SPECTRUM ANALYSIS IN SEISMOLOGY

by

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ABSTRACT

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Submitted to the Department of Geology and Geophysics on August 16, 1954 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

We first undertake to reacquaint the reader with the mathematical tools available for determining the spectrum of a function of time and also to establish the problem involved when one attempts to utilize these tools in estimating the spectrum. Adopting the method of estimation devised by Tukey (1949), a technique of computing spectra of a seismogram is suggested and actually applied to both earthquake and prospecting records. The procedure consists of computing spectra from hundreds of successive overlapping intervals and displaying them in the fashion of a contour map -- called, here, traveling spectra. The whole process is accomplished at high speed on the Whirlwind digital computer.

Application of the aforementioned process to several earthquake records obtained from the observatory in Weston, Mass. revealed some interesting dispersion curves for both Rayleigh- and Love- waves over Atlantic and continental paths. Those curves for surface waves which traveled the Atlantic path were strikingly similar whereas those curves for surface waves traveling over the continent were most dissimilar. This probably indicates a fair degree of lateral inhomogeneity in the continental portion of the earth's crust by way of comparison to the oceanic portion.

Similar techniques were also applied to some microseisms (seismograms recorded at Weston) due to a storm which existed over the Atlantic near the coast of New England. Our analysis revealed the existence of a resonance phenomenon suspected by Haq (1954) in addition to further corroborating the theory of the origin of microseisms proposed by Longvet-Higgins (1950) and recently studied by Haq.

Since the analysis of earthquake seismograms suggested the use of traveling spectra as a means of "picking" reflections, several trials were made on three prospecting records where reflections were both visible and invisible. The results were quite encouraging, for all types of reflections were put into evidence rather well.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF PLATES

LIST OF DIAGRAMS

LIST OF TABLES

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A STATEMENT OF THE PROBLEM

Throughout the existence of Seismology surprisingly little has been done to ascertain by means of exact analysis the frequency content of recorded seismic motion, in comparison to theoretical work which has taken place in this regard. The U. S. Coast & Geodetic Survey, we understand, has recently accomplished a good deal of work in this direction, as well as Housner, et al. (1953) who have devoted their attention to analysis of strong motion earthquakes.

Preceding their work, however, only a few papers have been written (Klotz (1918); Caloi (1939)) concerning the application of Fourier analyzing machines to portions of earthquake records. Jakosky, et al. (1952) have been engaged in frequency analysis of reproducable prospecting records (magnetic recordings), and as far as we know, their work and possibly that of a few companies in the petroleum industry (yet unpublished) is all the experimental research that has been projected in this direction in the field of exploration seismology.

It is not difficult to understand the dearth of work in analysis of this sort when one considers the enormous amount of labor that such investigation calls for. Only the advent of high speed computing machinery has made the undertaking of such analysis feasible.

As a matter of fact, it has only been in these recent years which have seen the perfection of these electronic computors that serious consideration has been given to the problem of calculating spectra. Possibly this is coincidence, but there is reason to believe that such study was motivated by the development of such machines. We believe that there are only two reports which concern this problem of spectrum calculation -- the original (and probably the major) work by Dr. J. W. Tukey (1948-9) and the recent work of D. T. Ross (1954).

In this thesis we have endeavored to apply Tukey's discoveries to the spectrum analysis of seismograms and to exploit as much as possible the high speed of M.I.T.'s electronic computor to that end.

Our purpose was not only to devise a means of seismogram spectrum analysis, but also to apply such analysis in several cases in order to obtain useful information concerning surface wave dispersion; microseisms; and direct, refracted, and reflected energy of earthquakes and explosions. It was also our hope that our choice of subject matter and manner of presentation would be such as to encourage future investigation along these and similar lines.

CHAPTER I

INTRODUCTION AND BACKGROUND

A. Introduction

In the following sections, an attempt is made to reacquaint the reader with the well known mathematical tools utilized in spectrum analysis; and in so doing an effort is made to categorize them -- in a fashion similar to that employed by Y. W. Lee (1950) -- according to assumptions necessarily attendant on the function to be analyzed.

We have also tried to put the problem of the actual process of computation of spectra into proper light in a section entitled "Concepts of the Spectral Window". It is hoped that, in consideration of these facts, the techniques we have employed in the analysis of seismograms will be brought into better perspective.

A method of analyzing seismogram spectra by determining frequency amplitudes and phase magnitudes of successive overlapping intervals is introduced. We also make known a novel way of displaying these spectra in three dimensional form and a method of accomplishing this at high speed on a digital computer.

B. General Harmonic Analysis

The problem of harmonic analysis is an old one and has developed after passing through many stages, including the use of periodograms, correlograms, and various computational techniques. Many of these methods had little foundation as far as mathematical rigor was concerned when they were first introduced, yet each constituted an advance over previous techniques. This is also true of the so-called auto correlation methods which began to exert some influence over twenty years ago but which have, in recent years, gained a new impetus from Prof. N. Wiener's work on time series. It was a result of the firm mathematical basis which Prof. Wiener has laid down that many engineering and scientific fields have recognized the excellence and versatility of this method and have consequently given more emphasis to it.

In firtue of the above remarks, we will attempt in the following discussion to bring forward the "old" and "new" formulae involved in spectral analysis. The purpose of this is to refamiliarize the reader with the mathematical detail which will be referred to in later sections, and to acquaint him with the contrast and similarity between the methods. We have then, depending on the nature of the functions involved, three general modes of spectral analysis.

Only two modes will be discussed here since we have applied only two of them in forthcoming sections. An excellent treatment of the third -- that which concerns analysis of random functions -- is given by Y. W. Lee (1950). The first of these methods to be discussed is the analysis of periodic functions; the second that of aperiodic functions.

a. Periodic Functions

According to Fourier, representation of any continuous periodic time function of period T_1 is given by

$$f(t) = \sum_{n=-\infty}^{n=+\infty} F(n)\, e^{in\omega_0 t} \qquad (1)$$

where ω_0 is the fundamental angular frequency $= \frac{2\pi}{T_1}$, $i^2 = -1$ and $F(n)$ is the complex line spectrum given by the expression

$$F(n) = \frac{1}{T_1} \int_0^{T_1} f(t)\, e^{-in\omega_0 t} dt \qquad (2)$$

$F(n)$ may also be looked at in the following way:

$$F(n) = |F(n)|\, e^{i\phi_n} \qquad (3)$$

where

$$|F(n)| = \left(Re[F(n)]^2 + Im[F(n)]^2 \right)^{\frac{1}{2}} \qquad (4)$$

and

$$\phi_n = Tan^{-1} \frac{Im[F(n)]}{Re[F(n)]} \qquad (5)$$

where

$$Re[F(n)] = \frac{1}{T_1} \int_0^{T_1} f(t) \cos n\omega_0 t \, dt \qquad (6)$$

and

$$\operatorname{Im}[F(n)] = -\frac{1}{T_1}\int_0^{T_1} f(t)\sin n\omega_0 t\,dt \qquad (7)$$

We can see the relation of the auto correlation function and its transform, the power density spectrum, to the harmonic analysis of periodic functions if we recall the definition of the auto correlation function $\phi_{11}(\tau)$

$$\phi_{11}(\tau) = \frac{1}{T_1}\int_0^{T_1} f_1(t)\, f_1(t+\tau)\,dt \qquad (8)$$

and expand $f_1(t + \tau)$ in a Fourier Series and resubstitute into $\phi_{11}(\tau)$ for it can be easily shown that

$$\phi_{11}(\tau) = \sum_{-\infty}^{+\infty} |F(n)|^2\, e^{in\omega_0\tau} \qquad (9)$$

We can see then that the spectrum of the auto correlation function is tantamount to the power spectrum of the function and further that $\phi_{11}(\tau)$ retains all harmonics of the given function but drops all phase angles.

If we specify $|F(n)|^2$ (the power spectrum) by $\Phi_{11}(n)$ we may further state

$$\Phi_{11}(n) = \frac{1}{T_1}\int_0^{T_1} \phi_{11}(\tau)\, e^{-in\omega_0\tau}\,d\tau \qquad (10)$$

In other words, harmonic analysis of the auto correlation functionof a periodic function f (t) yields the power spectrum of f(t).

If we let

$$\phi_{12}(\tau) = \frac{1}{T_1}\int_0^{T_1} f_1(t)\, f_2(t+\tau)\,dt \qquad (11)$$

be the cross-correlation function between $f_1(t)$ and $f_2(t)$ and it can be shown by a somewhat similar extension of the aforementioned formulae, that the cross power spectrum is

$$\Phi_{12} = \frac{1}{T_1} \int_0^{T_1} \phi_{12}(\tau)\, e^{-in\omega_o\tau}\, d\tau \tag{12}$$

In passing we should note that the cross power spectrum is in general a complex function, and consequently the cross-correlation function retains relative phase information, in contrast with the auto correlation which discards it. In addition it is also true that the cross-correlation retains only those harmonics which are common to both $f_1(t)$ and $f_2(t)$.

b. Aperiodic Functions

If we should consider aperiodic functions as being defined by

$$\int_{-\infty}^{+\infty} f^2(t)\, dt \qquad \text{Finite}$$

we may regard $f(t)$ as being capable of being represented over all time in the manner of the Fourier Integral

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega)\, e^{i\omega t}\, d\omega \tag{13}$$

where

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, dt \tag{14}$$

Such a manner of representation is, of course, valid in the case where

$$f(t) = 0 \qquad \begin{array}{l} \text{for} \quad -\infty < t < T_1 \\ \text{and for} \quad T_2 < t < +\infty \end{array}$$

$F(\omega)$ is the complex amplitude spectrum of $f(t)$. There is considerable similarity between the expression for this function and the complex line spectrum of the periodic function, eq. (2). Except for the lack of the multiplicative constant $\frac{1}{T_1}$ expressions for the magnitude, $F(\omega)$; the phase $\phi(\omega)$; Re $[F(\omega)]$; and Im$[F(\omega)]$ may be obtained by substituting the variable ω for n in equations 4 through 7 respectively.

It may be shown in a fashion somewhat similar to that of eq. (9) that,

$$2\pi \int_{-\infty}^{+\infty} F(\omega)\, F(\omega)\, e^{i\omega\tau}\, d\omega = \int_{-\infty}^{+\infty} f(t)\, f(t+\tau)\, dt \tag{15}$$

$$2\pi \int_{-\infty}^{+\infty} |F(\omega)|^2\, e^{i\omega\tau}\, d\omega = \int_{-\infty}^{+\infty} f(t)\, f(t+\tau)\, dt = \phi_{11}(\tau) \tag{16}$$

where $\phi_{11}(\tau)$ unlike $\phi_{11}(\tau)$ of periodic function may be expressed

$$\phi_{11}(\tau) = \int_{-\infty}^{+\infty} f(t)\, f(t+\tau)\, dt \tag{17}$$

and where $F(\omega)$ is defined by equation (14).

If we let

$$\Phi_{11}(\omega) = 2\pi\, |F(\omega)|^2 \tag{18}$$

then we have

$$\phi_{11}(\tau) = \int_{-\infty}^{+\infty} \Phi_{11}(\omega)\, e^{i\omega\tau}\, d\omega \tag{19}$$

and by Fourier Transform theory

$$\Phi_{11}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \phi_{11}(\tau)\, e^{-i\omega\tau}\, d\tau \qquad (20)$$

Since $\phi_{11}(\tau)$ and $\Phi(\omega)$ are even functions the foregoing are simplified to read

$$\phi_{11}(\tau) = \int_{-\infty}^{+\infty} \Phi_{11}(\omega) \cos \omega\tau \, d\omega \qquad (21)$$

$$\Phi_{11}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \phi_{11}(\tau) \cos \omega\tau \, d\tau \qquad (22)$$

The last equations express the fact that the auto-correlation function and the energy density spectrum of finite total energy are Fourier cosine transforms of one another.

Similar relations exist for cross-correlation of two functions of finite total energy.

C. Computation of Spectra

The time-honored process of Fourier Analysis has in recent years become of great importance. It has always been recognized as a method of expressing in a most elegant manner many natural phenomena, and with the advent of high-speed computing devices it has quite naturally received increasing attention.

Although at one time the Fourier Series approach saw great use, more recent applications have demanded Fourier Transform techniques. However well the Fourier transform with its attendant continuous frequency variable fits the actual case there have nevertheless been additional complications introduced with the transfer.

The following discussion is an attempt to give the reader an insight as to the nature of these complications as well as a resume of what has transpired in recent years in efforts to alleviate them.

a. The concept of "spectral windows"

There are usually three minor types of errors involved in the process of the Fourier transformation

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, dt$$

They are encountered in reading $f(t)$ and $\cos \omega t$, in forming their multiplication, and in integration. Their effects on the resultant answer are nevertheless quite small and may be stated in probabilistic form depending on the methods employed.

The major error in this process of transforming empirical functions arises because these functions are known only over some finite interval. Such is the case when it is desired to obtain the spectrum of an interval of a time series, for here the function must be regard~~ing~~ed as known over the interval in question and zero elsewhere. The effect on the computed transform is to superimpose relatively high amplitude oscillations on the correct transform. (This is the so-called Gibbs phenomenon. cf. Guillman, Carslaw)

The resultant spectrum, needless to say, is the incorrect one. Let us regard this incorrect spectrum (transform) of $f(t)$ as the correct spectrum of say $g(t)$. Coalescing the aforementioned errors into a weighting function $D(t)$ we may further regard $g(t) = D(t) \cdot f(t)$. Denoting the Fourier transformation process by which we obtained the incorrect spectrum by the operator F'' and the process by which we obtain the correct spectrum by the operator F' we obtain the equality,

$$F'[\, D(t)\, f(t) \,] = F''[\, f(t)] \tag{23}$$

Letting

$$F'[\, D(t) \,] = D(\omega)$$

and

$$F'[\, f(t) \,] = F(\omega)$$ (the correct spectrum)

and using the well-known fact that the transform of a product is the convolution of the factors involved we have

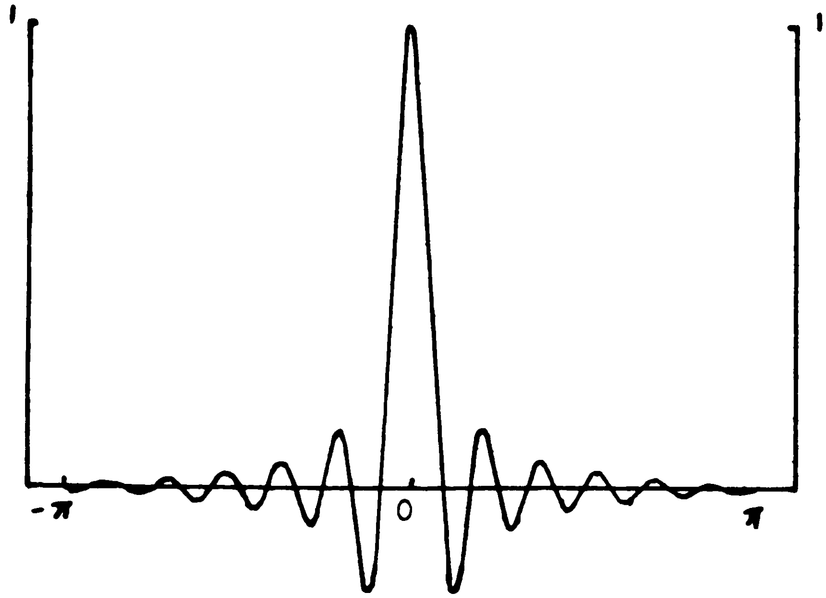
$$\int_{-\infty}^{+\infty} D(\omega-\xi)\ F(\xi)\ d\xi = F''[f(t)] \qquad (24)$$

where ξ has the same dimensions as ω .

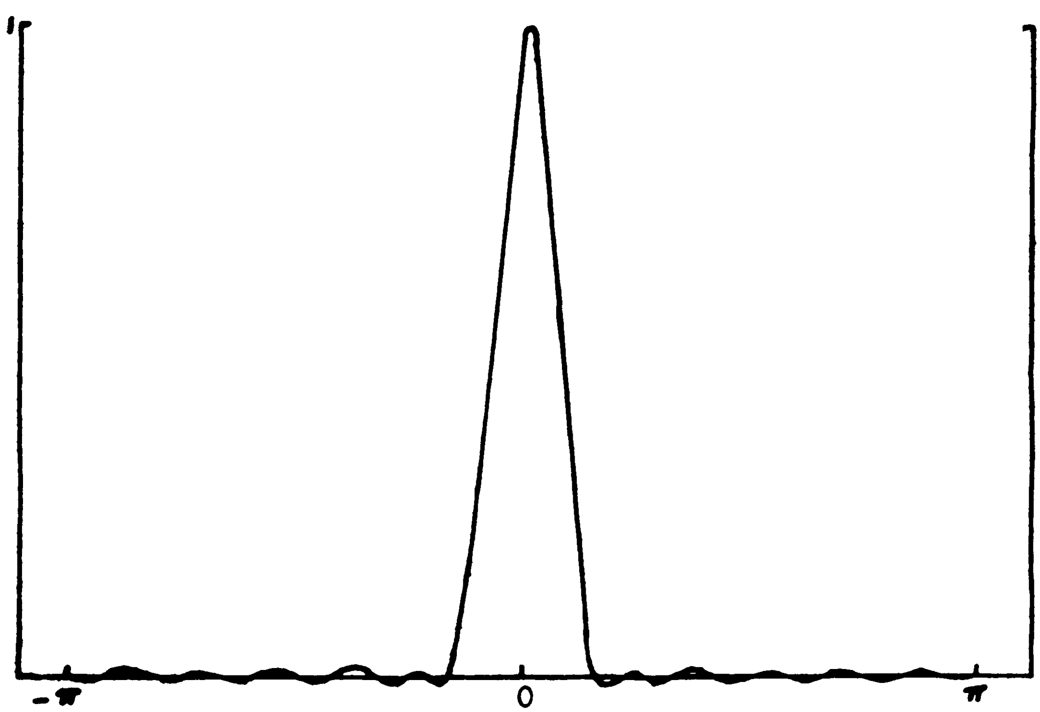
Hence, the value of the spectrum at ω_n which is estimated by the procedure of calculation decided upon, is obtained by taking the inverted transform of the weighting function, centering it about ω_n , and using this to weight the values of $F(\xi)$ over the entire range of ξ .

Henceforth, $D(\omega)$ will be referred to as the - "spectral window". Furthermore it is a function which is entirely independent of $f(t)$ and dependent only on the method of approximating the transform; and in turn it may be regarded as the measure of goodness of the method utilized.

We have drawn the $D(\omega)$ of $D(t)= 1$ corresponding to the most elementary transform process possible, in fig. (1) (referred to there is the "spectral window of L_0) and it may be seen from this that due to the "ripple" on either side of the $\omega = 0$ in the fig.) that a poor estimate will be obtained for the spectrum. Ideally $D(\omega)$ should equal the impulse function, for then the estimate would be the correct value. The best that can be done is to "smooth"



SPECTRAL WINDOW OF L_0
(The Transform of the Weighting
Function $D(t) = 1$)



SPECTRAL WINDOW OF U_0
(The Smoothed Transform of the Weighting
Function $D(t) = 1$)

Fig. (1)

out the ripple of the D (ω) so that the estimate may be obtained by averaging the correct spectrum over the narrowest band of frequencies possible. We have shown Tukey's success (1949) in this respect in the same fig. (1) by the configuration entitled "spectral window of U_0" (we will report on this later). Tukey accomplishes this smoothing by the use of an averaging process in forming the transform.

In the frequency plane the spectral window may also be looked upon as a weighting function.-- $W_\nu(\omega)$.
To establish this concept, which will be referred to later, the following development arises.

We may regard the spectrum estimate S'' as related to the correct spectrum S by the equality,

$$S''(\nu) = \int_{-\infty}^{+\infty} W_\nu(\omega)\, S(\omega)\, d\omega \qquad (25)$$

where ω and ν are of the same dimensions and ν indicates the scanning frequency. We have already seen that for even functions

$$S''(\omega) = \int_{-\infty}^{+\infty} D(t)\, f(t) \cos \omega t\, dt \qquad (26)$$

If we let

$$f(t) = \int_{-\infty}^{+\infty} S(\nu) \cos \nu t\, d\nu \qquad (27)$$

then

$$S''(\omega) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} S(\nu) \cos \nu t\, d\nu \right] D(t) \cos \omega t\, dt \qquad (28)$$

Reversing the order of integration eq. 28 becomes

$$S''(\omega) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} D(t) \cos\nu t \cos\omega t \, dt \right] S(\nu) \, d\nu \qquad (29)$$

Hence, the spectral window regarded as a weighting function is for cosine transformation

$$W_\nu(\omega) = \int_{-\infty}^{+\infty} D(t) \cos\nu t \cos\omega t \, dt \qquad (30)$$

For sine transformation

$$W_\nu(\omega) = \int_{-\infty}^{+\infty} D(t) \sin\nu t \sin\omega t \, dt \qquad (31)$$

Remembering that S'' depends on the interval length T we can now consider the error involved in calculating

$$S''(\omega, T) = 2 \int_0^T f(t) \cos\nu t \, dt \qquad (32)$$

instead of

$$S''(\omega, \infty) = 2 \int_0^{+\infty} f(t) \cos\nu t \, dt \qquad (33)$$

In order to formulate a comparison let us consider the case where $f(t) = \cos\nu t$ and $\nu = \omega$, for here the answer is known and a measure of the effect of integration over a finite interval may be arrived at. $D(t)$ here equals 1.

Here then

$$2 \int_0^T \cos^2 \omega t \, dt = T + \frac{\sin 2\omega T}{2\omega} \qquad (34)$$

and which when normalized equals

$$1 + \frac{\sin 2\omega T}{2\omega T} \qquad (35)$$

Graphically the comparison is

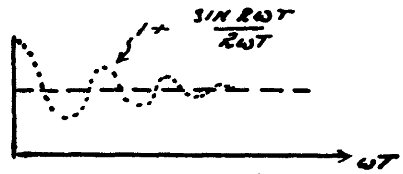


Diagram (1)

and we see that as the interval is lengthened the estimated value approaches closer and closer the correct value.

The above process where ω is kept constant and the error associated with the method is examined as T is varied, is, from examination of eq. 30 and 31, equivalent to measuring the goodness of the process in the frequency plane with the length of interval held constant. In other words if we took eq, 32 and did not vary the length of the interval as in diagram (1) we would have an expression equivalent to eq. 30, which expresses the value of our estimate of the spectrum in the frequency plane.

In the process of smoothing it is necessary to keep in mind both the window in the frequency domain and the corresponding weighting function in the time domain. Ross (1954) recognizes this in the development of a series of weighting functions $D_n(t)$, (reproduced in fig. (2) where n indicates the stage of smoothing, i.e. the greater n the more is the "ripple" of the spectral window diminished. He cautions

ROSS'S
NORMALIZED WEIGHTING FUNCTIONS

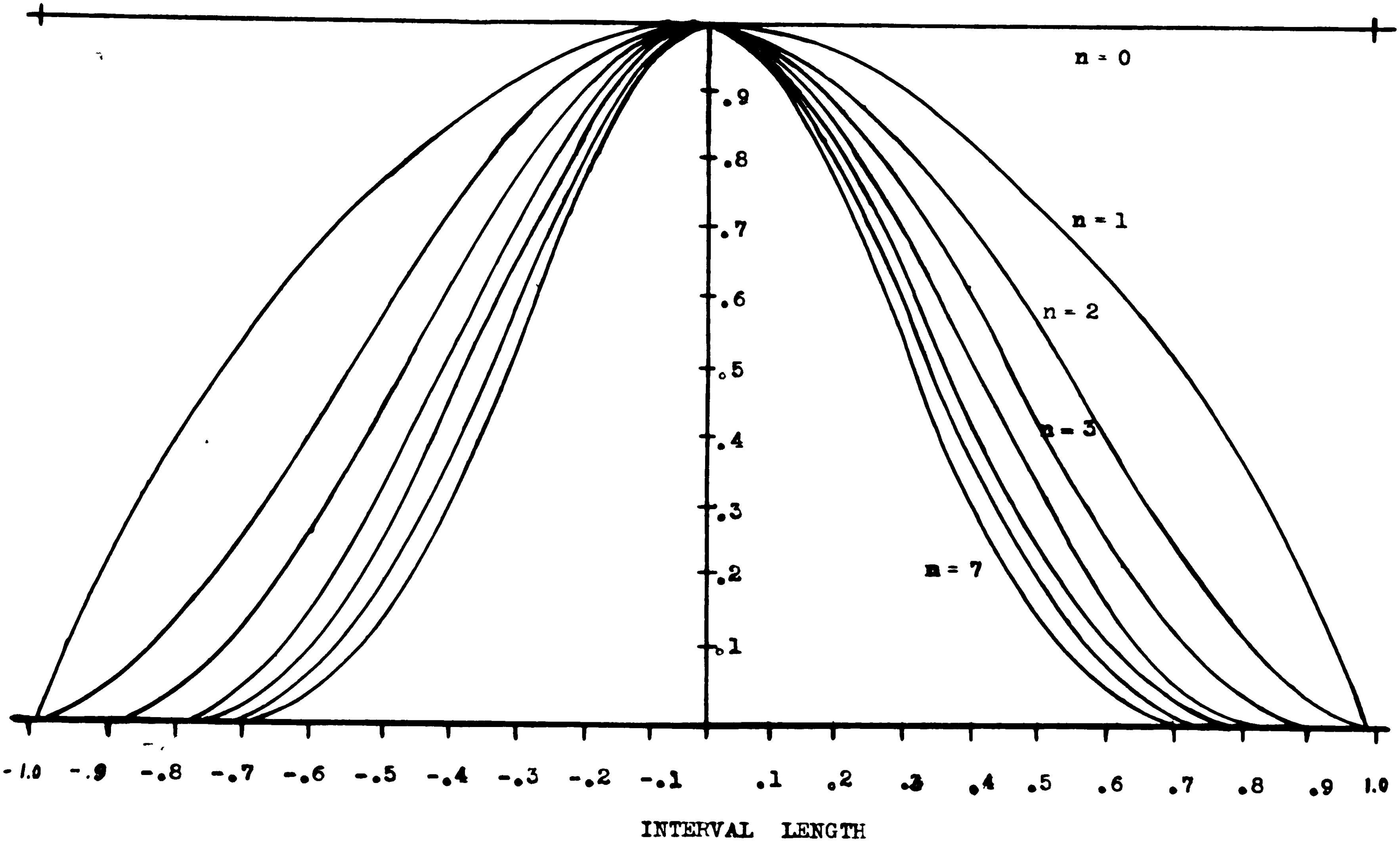


Fig. (2)

that -- if we take $D_n(t)$ for larger and larger values of n we, at each stage, essentially reject more and more the finite record of $f(t)$ for large t, and more and more emphasis is placed on values of f (t) for small t. The estimate of the spectrum may then be further from the truth than that obtained by using a smaller value of n.

It is clear that a balance must be achieved and this balance will be largely dependent on f(t). For instance

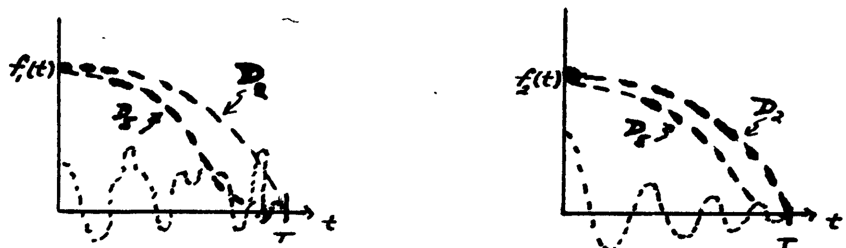


DIAGRAM 2

it is obvious that $f_2(t)$ can be smoothed more than $f_1(t)$ with little loss of information at large values of t and as a result a better spectr um is attained. In general we may say that the greater the degree of smoothing the better the spectrum estimate -- <u>provided</u> that the weighting function in the time domain associated with the process of smoothing in the frequency domain, does not reject a representative portion of the function being transformed.

The auto-correlation functionis, generally speaking, somewhat similar in configuration to $f_2(t)$ in diagram (2). If we should further consider $f_2(t) = \phi_{11}(\tau)$ where $\phi_{11}(\tau)$ is the auto-correlation of $f_1(t)$, we can easily see from our above discussion that transforming the auto-correlation of a function $f_1(t)$ will -- for the stage of smoothing involved-- give rise to a better spectrum than the process which entails transforming the function $f_1(t)$ itself.

(b) Estimation of the Power Spectrum

We should now like to outline the method of estimating the power spectrum of a discrete time series $x_1, x_2 \ldots x_n = f_1(t)$ utilized in our analysis of seismograms. This method consists of finding numerical approximations of the expressions for the auto-correlation function

$$\phi(\tau) = \int_{-\infty}^{+\infty} f(t)\, f(t + \tau)\, dt \qquad (36)$$

and the spectrum

$$\Phi(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \phi(\tau) \cos \omega\tau \, d\tau \qquad (37)$$

which hold for stationary time series. The following discussion of this method is quoted from Report #5 of the Geophysical Analysis Group of M.I.T.

"The auto-correlation coefficients of a time series are given by the sample serial products

$$R_p = \frac{1}{n - p} \sum_{i=1}^{n-p} x_i \, x_{i+p} \qquad (38)$$

The basic problem is to obtain an approximation to the spectrum from the serial products R_p $(p = 0, 1 .. m)$ for a given number m which is less than n.

"The observations x_i have the spacing h, which may be defined as one unit. Since periods less than two units will not be observable themselves, the effect on the estimation of the spectrum is to fold over the last part of the frequency scale where $(\frac{2\pi}{\omega} < 2)$ into that portion of the scale where $(\frac{2\pi}{\omega} \geqslant 2)$. This means of course that on ω scale the distribution of frequencies will now run from $-\pi$ to $+\pi$, and we shall confine our attention to this region. The reduced spectr um is thus defined to be a spectrum which involves frequencies which in magnitude are no greater than π. Thus the frequencies ω, $2\pi-\omega, 2\pi+\omega$ .... are treated as aliases of each other. Moreover, to use both positive and negative frequencies, equal power is required at $-\omega$ and $+\omega$.

"By choosing discrete values of angular frequency $\omega = \frac{s\pi}{m}$ $(s = 0,1,....m)$, we may perform numerical integration of equation by <u>the trapezoidal rule</u> and write

$$\Phi(\omega_s) = \Phi\left(\frac{s\pi}{m}\right) \approx L_s \qquad (39)$$

where

$$L_s = \frac{1}{\pi}\left[\frac{1}{2} R_o \cos 0 + \sum_{j=1}^{m-1} R_j \cos\frac{s\pi j}{m} + \frac{1}{2} R_m \cos s\pi\right] \quad (40)$$

By letting

$$r_j = R_j / R_o \qquad (j = 0,1,2,...m)$$

we have

$$L_s = \frac{R_o}{2\pi}\left[1 + 2\sum r_j \cos\frac{s\pi j}{m} + r_m \cos s\pi\right] \qquad (41)$$

"An approximate integration of $\Phi(\omega)$ from $-\pi$ to $+\pi$ by the trapezoidal rule yields.

$$\int_{-\pi}^{+\pi} \Phi(\omega)\, d\omega = 2\int_{0}^{+\pi} \Phi(\omega)\, d\omega \approx \frac{\pi}{m}\left[L_0 + 2\sum_{1}^{m-1} L_s + L_m \right] \quad (42)$$

Now by standard summation formulae, we find that

$$L_0 + 2\sum_{1}^{m-1} L_s + L_m = \frac{m}{\pi} R_0 \quad (43)$$

Thus, for all values of m, the area given by the trapezoidal rule is R_0, the serial product of lag zero. Hence, we see, once again that in the estimated spectrum the total power has been compressed into the interval $(-\pi,\pi)$."

(c). Estimation of the Amplitude Spectrum

In a fashion somewhat parallel to that outlined in the previous section, we desire now to outline our method for approximating the Fourier transform of

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\, e^{i\omega t}\, d\omega \quad (44)$$

In particular we wish to approximate $|F(\omega)|$ and $\phi(\omega)$ where

$$F(\omega) = |F(\omega)|\, e^{i\phi(\omega)} \quad (45)$$

and where in turn

$$|F(\omega)| = \left(Re[F(\omega)]^2 + Im[F(\omega)]^2 \right)^{\frac{1}{2}} \quad (46)$$

and

$$\phi(\omega) = Tan^{-1} \frac{Im[F(\omega)]}{Re[F(\omega)]} \quad (47)$$

From section B we can see that calculation of the above relies on estimating the integral representations of Re $[F(\omega)]$ and $Im[F(\omega)]$ which are

$$\begin{aligned} Re[F(\omega)] &= +\int_{-\infty}^{+\infty} f(t)\cos\omega t\,dt \\ Im[F(\omega)] &= -\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt \end{aligned} \qquad (48)$$

Here we considered $f(t)$ a discrete time series of spacing h.

$$f_i(t) \approx f(t_o),\ f(t_o+h),\ \ldots.\ f(t_o+ih) \qquad (i = 0,1,2,\ldots m)$$

and we also considered

$$Re[F(\omega)] = Re[F(\frac{s\pi}{m})] \approx M_s \qquad (a)$$

$$(49)$$

and

$$Im[F(\omega)] = Im[F(\frac{s\pi}{m})] \approx -N_s \qquad (b)$$

where $s = 0,1,2,\ldots.m$

As before we numerically integrate (49a) and (49b) by the trapezoidal rule in the familiar fashion.

$$M_s = +\left[\frac{1}{2} f_o \cos 0 + \sum_{j=1}^{m-1} f_j \cos\frac{s\pi j}{m} + \frac{1}{2} f_m \cos s\pi\right]$$

$$-N_s = +\left[\frac{1}{2} f_o \sin 0 + \sum_{j=1}^{m-1} f_j \sin\frac{s\pi j}{m} + \frac{1}{2} f_m \sin s\pi\right]$$

We point out at this time that the weighting function, D(t), employed in the estimation of the L_s, M_s, and N_s is 1, and as a consequence the estimate is quite far from the truth, which fact is at once evident from the corresponding spectral window, depicted in fig. (1). To overcome this drawback we employed a method of smoothing which we describe in the subsequent section.

d.. A Method of Smoothing

Dr. T. W. Tukey of Bell Telephone Laboratories and Princeton University (1948-1949) has made what is probably the major contribution to the study of spectrum estimates in recent years. His work, which has undergone several revisions and additions, consists mainly of two studies.

The first part of his work (1949), which was accomplished with the assistance of R. W. Hamming, consists in choosing and applying an improved weighting function which uses the familiar property of Fourier transforms: that the transform of a trigonometric polynomial is a function consisting of equally spaced impulses of unequal strength. The process of convolving the transform of such a polynomial (e.g. a cosine arch) becomes, for digital calculations, the applicationof a running weighted mean or smoothing formula.

Tukey's work, besides being located in the above cited reference may be found beautifully summarized in Report #5 of the Geophysical Analysis Group, M.I.T. His formulae, which we have used in smoothing our estimated spectra of seismograms, calculated in the second portion of this thesis, are listed below for the reader's convenience.

The Tukey-Hamming smoothed estimate, U_s , of the power spectrum density and the cosine and sine transforms of f(t) is given by,

$$U_s = .23X_{s-1} + .54X_s + .23X_{s+1}$$

where X is equal to the L, M, and N of sections b and c.

Since U_o and U_m respectively involve X_{-1} and X_{m+1}, which have not been defined, we set $X_{-1} = X_1$ and $X_{m+1} = X_{m-1}$

This amounts to

$$U_o = .54X_o + .46X_1$$

$$U_m = .54X_m + .46X_{m-1}$$

When $X = L$ we see that because of the identity

$$U_o + 2\sum_1^{m-1} U_s + U_m = L_o + 2\sum_1^{m-1} L_s + L_m$$

the smoothing process is area preserving, and hence the total area in the estimated spectrum is given by R_o. See equation (38). This conclusion is Parseval's Theorem.

The frequency in c.p.s. for which the various line spectra U_s have been calculated are $\nu = \frac{s}{mh}$ $(s = 0,1,2,\ldots m)$ where m is the total number of lags or points, and h is the time spacing between points of the discrete time series.

D... <u>Some Considerations of Numerical Fourier Analysis</u>

There are two distinct ways of regarding the numerical method in general -- first as an approximation to the integrals for each line spectrum, and secondly as a process of curve-fitting. It is wise to keep both in mind when applying or interpreting the results.

As may have already been noticed, our approach to calculating the spectrum of an interval on a seismogram has been to choose a fundamental frequency, determined by the length of the interval T_i, and the time spacing between consecutive points, h and then to compute the line spectra of this frequency as well as its odd and even harmonics. The approximation of the intervening frequencies involved in the transform was accomplished by the smooth curve connecting these calculated points. Essentially then, our calculated values were derived in the manner which would have been employed if the interval T_i were considered representable by a Fourier series.

Rieman's theorem states that the Fourier representation is unique. The theorem, however, applies only when no limit is imposed upon the number of terms which may be included; if any arbitrary limitation is imposed, the result of the theorem is not necessarily valid.

In the following discussion we propose to ascertain the restrictions of and the reliability of our analysis when a limitation is imposed on the quantity of data. Since the remarks made will pertain to our calculated values, we may for the purposes of this discussion regard $f(t)$ capable of being expressed in the manner of a Fourier series

$$f(t) = C + \sum_{n=1}^{n=k} a_n \cos n\omega_o t + \sum_{n=1}^{n=k} b_n \sin n\omega_o t \qquad (54)$$

where

$$\left.\begin{aligned} a_n &\approx \frac{2}{N} \sum_{r=1}^{r=N} f_r(t) \cos(n\omega_o t)\, r \\ b_n &\approx \frac{2}{N} \sum_{r=1} f_r(t) \sin(n\omega_o t)\, r \end{aligned}\right\} N = \frac{T_1}{h} \qquad (55)$$

Let us for the moment consider an example of a function periodic in 2π radians, for which we have values at $t = 0, \frac{\pi}{2}, \pi$ and $\frac{3\pi}{2}$ denoting these values by y_o, y_1, y_2, y_3

$$\begin{aligned} y_o &= f(0) = C + \Sigma a_k \\ y_1 &= f(\tfrac{\pi}{2}) = C + \Sigma a_k \cos \tfrac{k\pi}{2} + \Sigma b_k \sin \tfrac{k\pi}{2} \\ y_2 &= f(\pi) = C + \Sigma a_k \cos k\pi \\ y_3 &= f(\tfrac{3\pi}{2}) = C + \Sigma a_k \cos \tfrac{k3\pi}{2} + \Sigma b_k \sin \tfrac{k3\pi}{2} \end{aligned} \qquad (56)$$

Consider the effect of summing these four values

$$A_o = \sum_{n=1}^{3} y_n = 4C + \Sigma a_k D_k + \Sigma b_k E_k$$

where

$$\begin{aligned} D_k &= 1 + \cos \tfrac{k\pi}{2} + \cos k\pi + \cos \tfrac{k3\pi}{2} \\ E_k &= \sin \tfrac{k\pi}{2} + \sin \tfrac{k3\pi}{2} \end{aligned} \qquad (57)$$

A_o depends on the value of D_k and E_k . Now if k is a multiple of 4 , say $4X$ where X is an integer we find

$$D_{4X} = 4 \qquad E_{4X} = 0$$

$$D_{4X-1} = 0 \qquad E_{4X-1} = 0$$

and similarly $D_{4X-2} = E_{4X-2} = D_{4X-3} = E_{4X-3} = 0$

Substituting into the above expression for .. A_o

$$A_o = 4[\, C + \sum_X a_{4X} \,] \qquad (58)$$

Consider the effect of performing the summation

$$A_1 = y_o \cos 0 + y_1 \cos \frac{\pi}{2} + y_2 \cos\pi + y_3 \cos \frac{3\pi}{2}$$

$$= y_o - y_2$$

$$= \Sigma\, a_k\, [\, 1 - \cos k\pi \,]$$

$$= 2\Sigma\, a_{2X-1}$$

Similarly the summation:

$$B_1 = y_o \sin 0 + y_1 \sin \frac{\pi}{2} + \ldots\ldots \qquad (59)$$

$$= y_1 - y_3$$

$$= \Sigma\, a_k\, D_k' + \Sigma\, b_k\, E_k'$$

where

$$D_k' = \cos \frac{k\pi}{2} - \cos \frac{k3\pi}{2}$$

$$E_k' = \sin \frac{k\pi}{2} - \sin \frac{k3\pi}{2}$$

If $k = 4X$ $\qquad$ $k = 4X-2$

$$D_{4X}' = \cos 2\pi - \cos 6\pi = 0 \qquad D_{4X-2}' = 0$$

$$E_{4X}' = 0 \qquad E_{4X-2}' = 0$$

$k = 4X-1$

$$D_{4X-1}' = \cos \frac{\pi}{2} - \cos \frac{3\pi}{2} = 0$$

$$E_{4X-1}' = - \;\;\text{"}\;\; + \;\;\text{"}\;\; = -2$$

$k = 4X-3$ $\qquad D'_{4X-3} = \cos\frac{3\pi}{2} - \cos\frac{\pi}{2} = 0$

Substituting into (59) $\qquad E_{4X-3} = -\ " \quad + \quad " = 2$

$$B_1 = 2\,\Sigma\,[b_{4X-3} - b_{4X-1}]$$

Proceeding in a manner similar to that above

$$\begin{aligned} A_2 &= y_0 \cos 0 + y_1 \cos \pi + y_2 \cos 2\pi + y_3 \cos 3\pi \\ &= y_0 - y_1 + y_2 - y_3 \\ &= 4\,\Sigma\, a_{4X-2} \end{aligned}$$

$$\begin{aligned} B_2 &= y_o \sin 0 + y_1 \sin \pi + \ldots . \\ &= 0 \end{aligned}$$

The results are expanded below:

$$\begin{aligned} A_o &= [4C] + 4a_4 + 4a_8 + \ldots .. \\ A_1 &= [2a_1] + 2a_3 + 2a_5 + \ldots .. \qquad (60) \\ B_1 &= [2b_1] - 2b_3 + 2b_5 - 2b_7 + \ldots \\ A_2 &= [4a_2] + 4a_6 + 4a_{10} + \ldots ... \end{aligned}$$

We note that if the variation contains harmonics higher than the 2nd, no information concerning the coefficients of <u>any</u> harmonics is afforded by the equations (60).

From the above example and others one can see that the following rule must be observed in numerical Fourier Analysis.

The number of ordinates must exceed twice the reference number of the highest harmonic present in the <u>unlimited</u> and

This restriction has also been determined by Goldman ('53) by a different procedure. The above discussion, however, points out what occurs when the restriction is <u>not</u> met.

Let us for example consider a periodic function which is made up of forty harmonics of a given fundamental frequency. The above result states that at least eighty (80) discretely spaced points of the function must be used if one wishes to ascertain the line amplitude spectrum of all forty harmonics. It does not state that although one may only be interested in the first twenty (20) it is sufficient to use forty points. Eighty must be used for otherwise, as already pointed out in our analysis, no correct information will be achieved.

In practice one assumes the function he is dealing with as made up of a finite number of harmonics, which for practical purposes is certainly valid. But since the exact number is usually unknown, the only alternative the investigator has is to experiment with sets of points, which have been read at various spacings, until two calculated spectra are similar. At this point it can be assumed that the above criterion is met.

E... Method of Displaying the Estimated Spectra

In the subsequent analysis of seismograms, whether they be prospecting or earthquake records, it was decided to break up the entire trace into intervals T, which overlap each other by 75%. The length of T is unspecified, but is assumed to comprise significant frequency information and to be subject to Fourier integral representation. In the following investigation the length of T found to be most practical was 40h, where h is the time spacing of the discrete time series .

In each interval an analysis is performed yielding either a smoothed estimate of the amplitude and phase spectrum or a power spectrum to be associated with that interval. Instead of plotting and evaluating each curve separately the information derived is presented in the fashion of contour maps, where the amplitude phase and power density is to be represented in relief on a map on which is plotted frequency vs time according to the diagram below

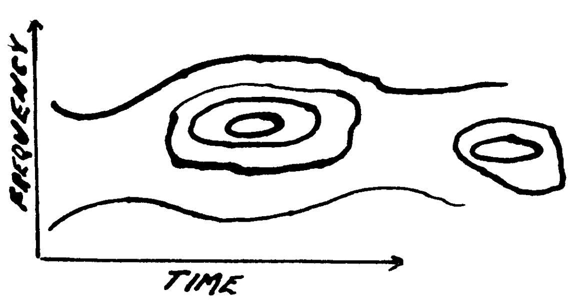
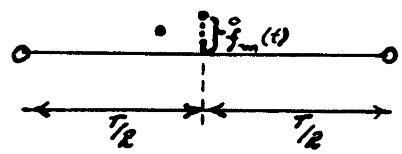


Diagram (3)

Inasmuch as many spectra are presented in this way the time dimension, though "blurred" because of interval overlap, has in a loose sense been preserved. The time which we have associated with the spectrum of each interval has been that time associated with $f_m(t)$, where $f_m(t)$ is the midpoint reading of the interval in question.



This time will henceforth be referred to as "center time", and the contour representation of spectra of overlapping intervals as "travelling spectra".

The computation and plotting which such an undertaking involves is enormous and the time which would necessarily be expended in such an operation would be such as to discourage it at the outset. However, the entire job of computing and contouring has been accomplished at high speed in the Whirlwind Electronic Computor. The fruits of this computation constitute the second part of this report.

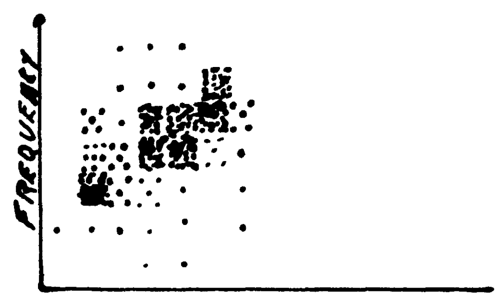
The mechanics of analyzing the spectra have already been desribed. The actual process of contouring has been accomplished by means of a density plot routine devised by Dr. S. M. Simpson of M.I.T. In this instance the dimension of "height" is brought about by the relative brilliance of a given square area on an oscilloscope screen.

In the case of amplitude or power density spectra all values in twenty-four consecutive spectra are normalized to the maximum value occurring in all these twenty-four. Each value is then associated with a number of spots to be focussed on a predetermined square area on the oscillioscope.

The greater the magnitude of the amplitude or power the more densely plotted are the spots.

These predetermined areas are arrayed in a "checkerboard" fashion in such a manner that "center time" is plotted horozontally (along the abscissa) and frequency is plotted vertically (along the ordinate). For a permanent record a photograph is taken of the density plot as it occurs.

The clarify the language, the following diagram constitutes a negative of a fictitious density plot.



Interval - Center-time

Diagram (4)

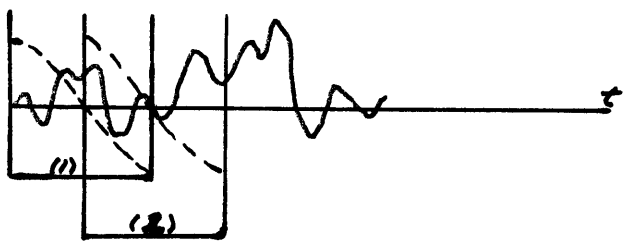
The density plots of the phase spectra are very similar to those described above for amplitude and power spectra. In this instance the only difference lies in that all phase angles are confined to the range of 0 to 360 , and the density of the spots are respectively less to great.

Each photograph of twenty-four overlapping intervals are connected in a consecutive fashion to give rise to a density plot of the entire seismogram trace. Not only does such an approach give tise to a neat, concise presentation of hundreds of seismogram spectra, but it also affords to the seismologist an accurate method of determining frequency and phase changes with time. In addition such a method enables easy comparison of frequency and phase changes with distance and with different component seismographs.

At present we will not be specific in their use or evaluation but will defer such comment to that time when actual cases are investigated.

F... The Phase Correction

In the calculation of the component frequencies of each overlapping interval τ, we found at the outset that, for the sake of ease and speed in digital computation, it would be best to scan each interval with harmonic frequencies which had a zero degree phase shift with respect to the beginning of each interval. Thus for the determination of the phase of the first harmonic we would scan the intervals in the following fashion for determination of $R_E[F(\omega)]$



For determination of the $I_m[F(\omega)]$

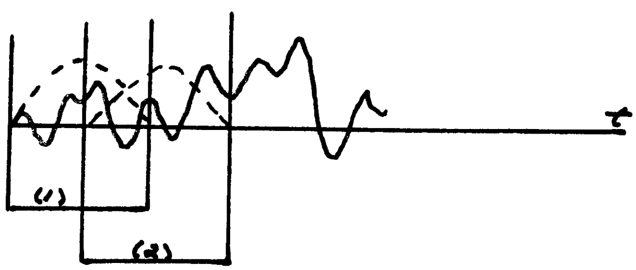


Diagram (5)

It may be seen that the phase $\tan^{-1}\frac{I_m}{R_E}$ calculated for such a harmonic will on a traveling spectrum basis be rather arbitrary depending on the interval and degree of overlap employed.

Regardless then of the interval we would like, however, to scan with a frequency which is essentially stationary in time, in order that the calculated phase would have some relationship with those calculated for other intervals and other frequencies. We would have then for computation of

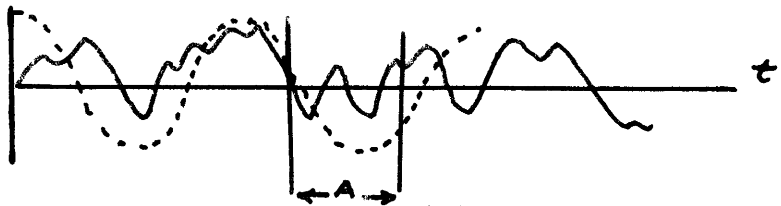


Diagram (6)

A similar diagram may be drawn for the intended calculation of

To accomplish this, one merely has to calculate the phase for each harmonic frequency employed (assuming 0^{o} phase shift for the interval in question), and then to subtract the phase angle by which the interval under examination shifts the scanning frequency from the same frequency stationary in time. For instance, suppose we wish to determine the phase relationship between the fundamental scanning frequency curve III and the same component frequency of $f(t)$ in interval A in diagram (7). We would first scan with the cosine arch, curve I, to determine Re $F(\omega)$ and with the sine arch, curve II, to determine $\operatorname{Im} F(\omega)$. Both curves exhibit 0^{o} phase with respect to interval A. The phase relationship between the component frequency of $f(t)$ and curve I is given by eq. (20). The phase relation we desire is then determined by subtracting from this calculation.

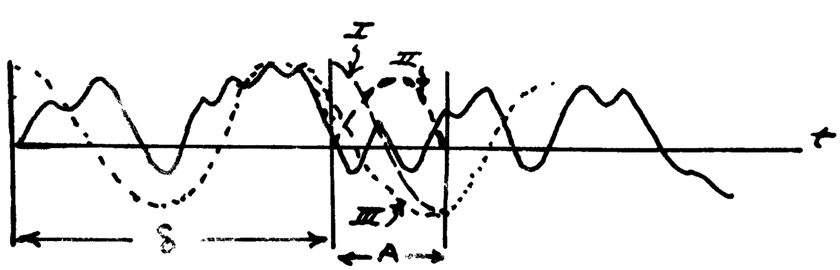


Diagram (7)

It may be easily shown that the phase shift δ which must be subtracted is dependent on the scanning frequency and the position of the interval. For intervals which overlap by 75%- the procedure that predominates in our investigation -- it may be further shown that this necessary phase shift δ is:

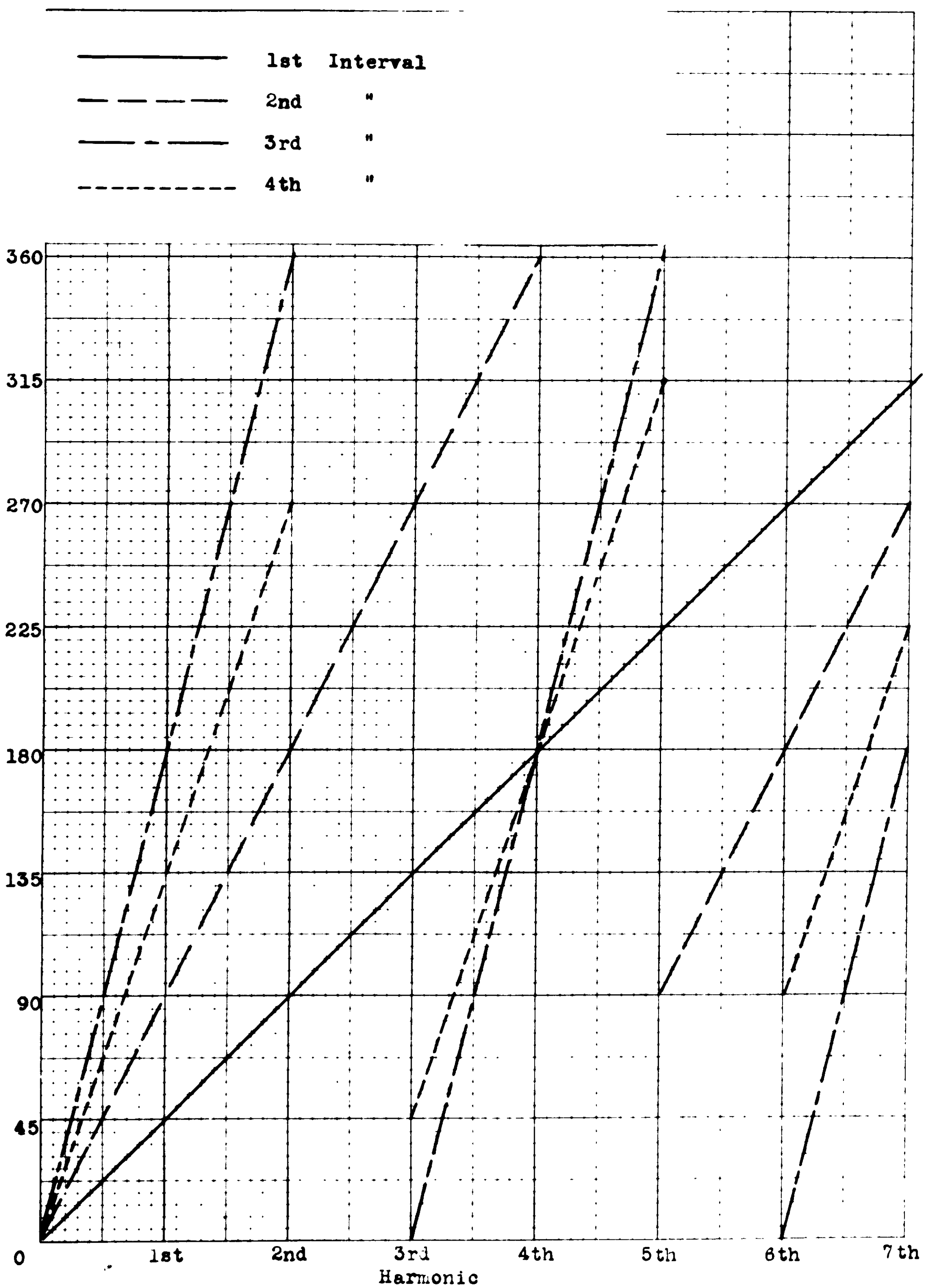
$$\delta = 45^\circ \, r \, s$$

where $r = 0,1,2,\ldots\ldots. m$ and denotes the harmonic .

and where $s = 0,1,2,\ldots\ldots m$ and denotes the interval.

From the nature of the overlap utilized it becomes evident that this δ , if looked at in the phase range 0° to 360° is repetitive for every eight harmonics and for every eight intervals, i.e. for $r = 0,1,..7$and for $s = 0,1,..7$. The same is true for $r = 8,..15$ and $s = 8\text{-}15$etc.

This phase shift is shown graphically for $r = 0,1,..7$ and for $s = 0,1,.7$in figs. (3) and (4). This phase shift which is then superimposed on the traveling phase spectrum, which is determined from scanning frequencies shifted zero degrees for every interval is displayed in fig. (5). The actual densities displayed for the various phases are here, not the true ones, but are nevertheless exemplary of the pattern.



1st Interval
2nd "
3rd "
4th "
360
315
270
225
180
135
90
45
0
1st
2nd
3rd
4th
5th
6th
7th
Harmonic
Phase Correction in Degrees

Fig. (3)

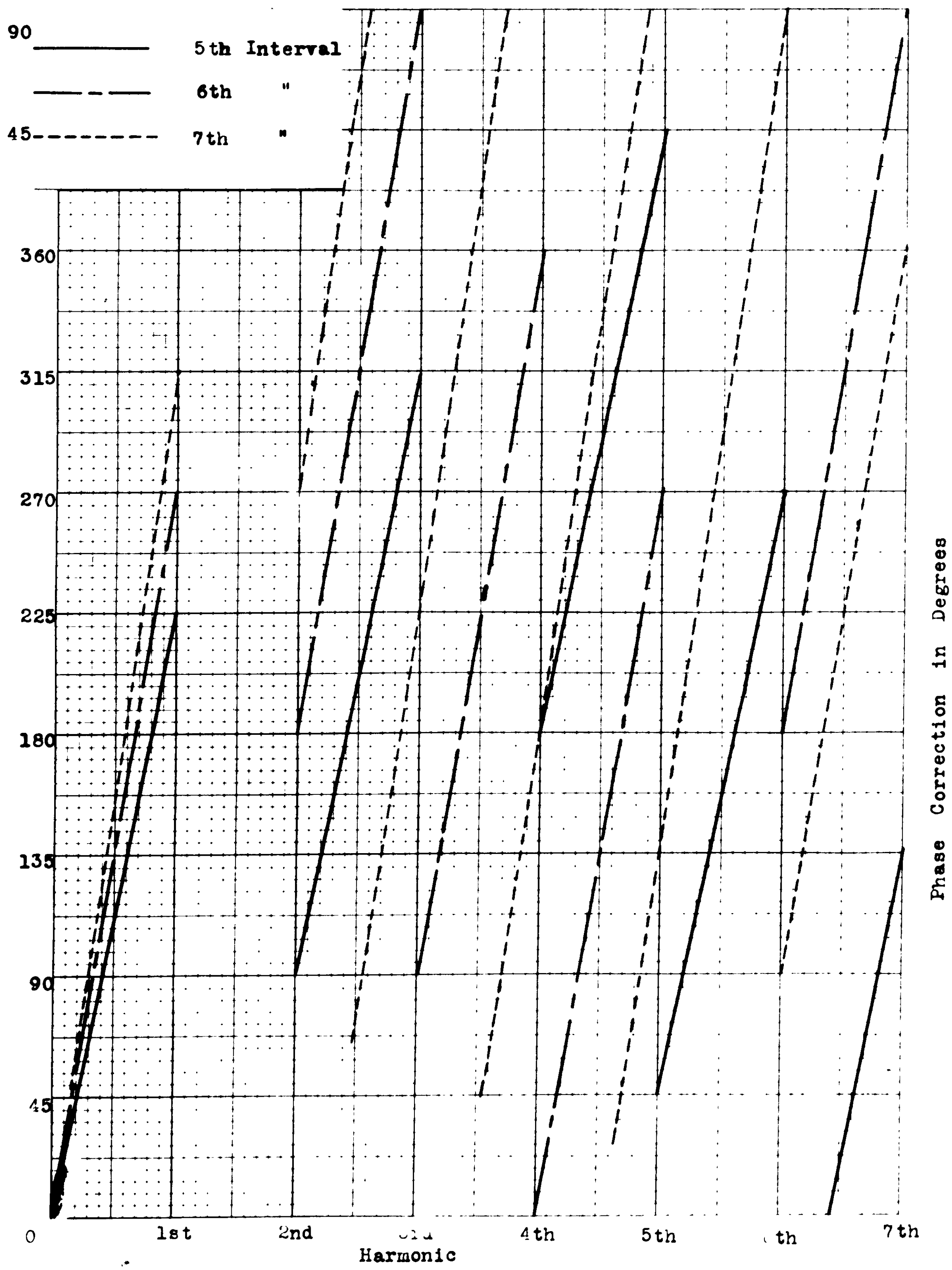
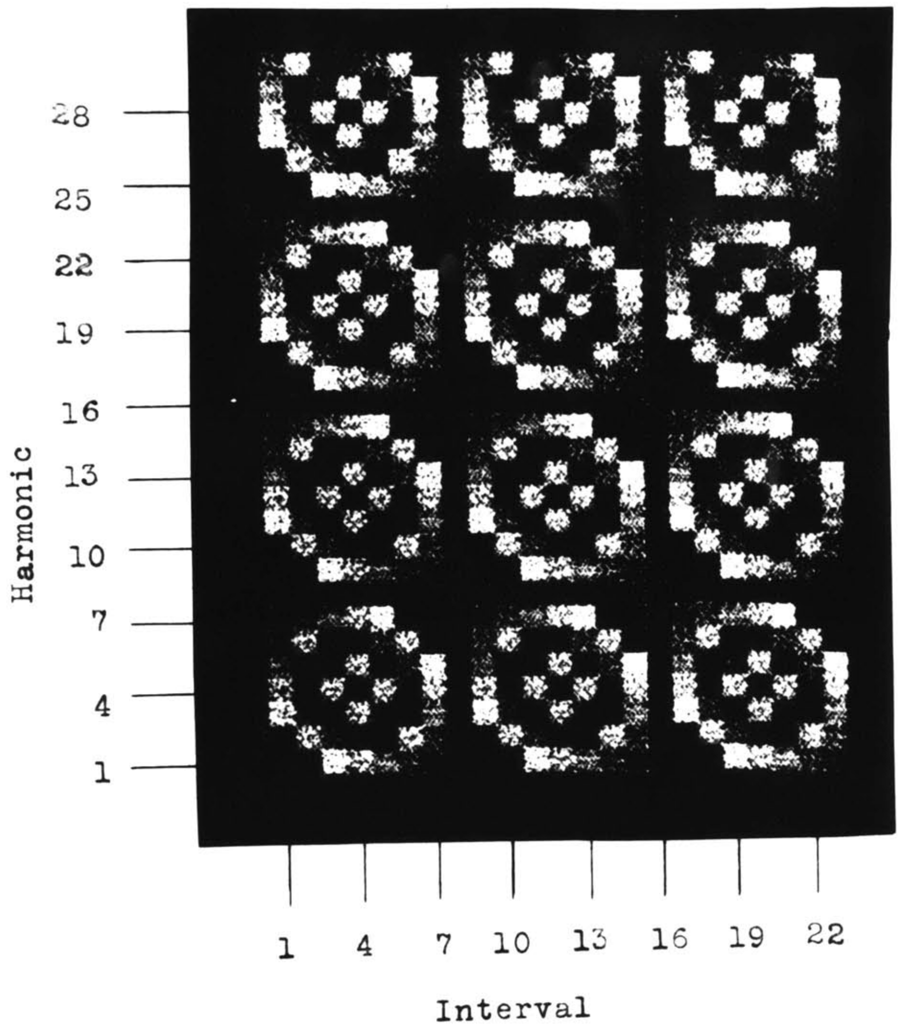

90
5th Interval
6th "
45
7th "
360
315
270
225
180
135
90
45
0
1st
2nd
4th
5th
7th
Harmonic
Phase Correction in Degrees

Fig. (4)



A Display of the Phase Correction Superimposed on the Traveling Phase Spectra Calculated from Scanning Frequencies Shifted 0 Degrees.

Fig. (5)

CHAPTER II

APPLICATION OF SPECTRUM ANALYSIS IN SEISMOLOGY

A... Introduction

In this section we have presented, in what we believe to be a rather unique manner, the results of our spectrum analysis of some seismic records. Unfortunately, we could cover only a few aspects in earthquake and exploration seismology. Factors of time and money and the fact that our undertaking was partially a test prevented further computation. It is our hope that the few cases considered here are both representative and interesting.

In passing we would like to point out --- at least for the sake of general interest --- that the calculation and plotting of spectra presented in the following sections would, if undertaken with the ordinary means available, occupy a man working full time for approximately ten years.

B... <u>A Study of Surface Wave Dispersion</u>

Determination of the earth's crustal structure have been greatly assisted by studies of the reflected and refracted waves of depth charges, quarry blasts, and near earthquakes; by studies of surface wave dispersion; and by considerations of amplitude ratios of direct and surface reflected longitudinal waves. Application of surface wave dispersion to this end has received, it seems, renewed impetus in recent years. Both theoretical study, which has been devoted to the betterment of this technique, and improvements in observation have been the motivating influence.

Generally speaking there are two ways for evaluating the amount of dispersion of observed surface waves. One is to determine the period and speeds of conspicuous or of first surface wave arrivals from the records of any number of earthquakes and stations, sort them as to type and path, and plot the data points for comparison with theoretical curves computed from some assumed type of crustal layering. The second method is to determine the pertinent data for successive waves in a surface wave group from a single seismogram and plot these data for comparison with theoretical curves. Both have been used extensively although the latter is to be preferred, since it supplies a dispersion curve consistent with respect to path and does not entail the difficulties encumbant with respect to choice of what to measure in the former method.

Whichever of the two methods is employed various notions prevail as to the choice of group, but only one method exists in the actual measurement of the group's period, i.e. an assumption is made that what is to be measured is a sinusoid and then what is assumed to be the period is measured with dividers, and a time scale of some sort. This method may be regarded as sufficient at the initial portions of the Rayleigh or Love train where the record usually exhibits long period sinusoidal motion. However, as time proceeds the sinusoidal character diminishes and at certain times is completely absent. Present techniques of measurement then can hope to be accurate only over a limited range of the surface wave record.

Furthermore, theoretical considerations have shown that it is conceivable for two or more groups to arrive at a station at the same time depending on the layering configuration of the earth's crust through which the waves have passed. Since such observation is beyond the scope of existing methods, comparison with theoretical dispersion curves for assumed layering is greatly hampered.

Our proposed technique of "traveling spectra" analysis, comparatively speaking, affords a more accurate measurement of the period of a particular group and also exhibits change of period in time. The plotting of the spectra of overlapping intervals according to density not only automatically affords a travel time curve of the dispersing trains of one quake but will also yield a more complete observation than has hitherto been offered. Furthermore such a manner of analysis could also be looked upon as a standard by which to establish actual dispersion, since it appears from the diversity of method and scatter of "observations" of seismologists that some sort of standardization is desirable.

We have in our study chosen five earthquakes recorded at Weston, Massachusetts. In two of these an Atlantic Ocean path predominates, in two others the path is entirely continental (U.S.), and in the fith approximately half the path is over the Pacific Ocean and half over the United States. The foci of all of these quakes were of normal depth. Only the long period Benioff records were used.

Case I -- Atlantic Path

(1) Data Analyzed -- Methods Employed

The pertinent data for the two quakes studied here were derived from the U.S.Coast and Geodetic Survey and Jesuit Seismological Assn. respectively.

Quake (a) Date- May 31, 1953

Epicenter -- N.Coast Dominican Rep.(20 N 70½ W)

Time -- 19:58: 35 G.S.T.

Magnitude -- 7 (Pas); 7½ (Berk)

Epicentral Distance -- 2500 kms; 22.5°

Quake (b) Date- Aug. 15, 1941

Epicenter -- "Atlantic Ocean" (19° N 27° W)

Time -- 06: 09; 00 G.S.T.

Magnitude

Epicentral Distance -- 2850 kms; 43.65°

Photostatic reproductions of these quakes may be found on plates (1) and (3)

For quake (a) data points-or trace amplitudes-- were read from the seismogram every .446 seconds over the interval marked "A" on the traveling spectra of this quake (plate 2) and every 1.78 seconds over the interval marked "B" on this plate. For quake (b) data points were read at every 1.78 seconds.

The traveling spectra density plots of the spectra computed for quake (a) may be found on plate (2). In region "A" spectra were computed from 80 point intervals; in region "B" spectra were computed from 40 point intervals. In "A" n-harmonic frequencies of .014 C.P.S.were calculated ($n = 0,1,2,3, ----40$) per interval. A frequency scale appears at the left margin for this region. For region "B" n-harmonic frequencies of .007 C.P.S. were calculated per interval ($n = 0,1,2,-40$); a frequency scale appears at the right of plate (2) for this region. The traveling spectra computed for quake (b) are found on plate (4). For this quake harmonic frequencies of .007 C.P.S. were calculated for intervals consisting of 40 points ($n = 0,1,2,3, -- 40$).

In all instances our group velocity curves were obtained by drawing a smooth curve through the spectral density maxima which the traveling spectra depict for the various groups of waves involved in the Love and Rayleigh trains, and reading choosing those times where the curve intersected the frequency coordinate lines. Unfortunately the dispersion of the Love waves was not observed for quake (a) which fact may be due to the short distance involved and to the nature of the path (Wilson and Baykal 1948 have observed this for Love waves in their studies) -- although in retrospect, it may be due to our not overlapping intervals in region "A" by a sufficient percentage. At any rate our observations may be found tabulated in Tables I - III and depicted in figures (6) - (8).

For the purposes of comparison we have included in fig. (6) the observations of Wilson & Baykal (1948) of the dispersion of Rayleigh waves across the Atlantic.

The theoretical curve recently calculated by Jardetzky & press (1953) is also reproduced in fig. (7). In this latter curve the layer of sediment and water were considered. The basement layering for this curve is $H_1 = H_0$ $H_2 = \infty$; $\alpha_0 = 1.52$ KMS/sec.; $\alpha_1 = 6.9$ kms / sec.; $\alpha_2 = 8.1$ kms / sec.; $\rho_1 = 2.67\rho_0$ $\rho_2 = 3.0\rho_0$ where α is the compressional wave velocity, H is the thickness, ρ the density, and the subscript the layer in question. Such a curve is according to these authors experimentally indistinquishable from the case where $H_1 \ne \infty$ $\rho_1 = 3.0\rho_0$; $\alpha_0 = 1.52$ kms / sec.; $\alpha_1 = 7.90$ kms / sec.

We have also on fig. (8) reproduced two theoretical curves for Pacific Love wave dispersion, which have been recently published by Evernden (1954). The fig. (8) contains the pertinent assumptions involved β and ρ refer as usual to the shear wave velocity and density in gms / cm^3 respectively).

(2) Discussion

The dispersion observed for Rayleigh waves over the Atlantic is very similar for the two paths studied, although the "Atlantic Ocean" quake gives values of velocity which are somewhat lower for the higher periods (50-70 sec.) and the lower periods (11-18 sec.)

The lower values in the latter range in quake (a) occur on the N-S component. Our value for 10.1 sec. - paralleling the Dominican Republic Quake is questionable and we have indicated this on our plate.

If we assume that the land path in both cases was 450 kms -- the distance from Weston to the 2000 fathom line - and that the speed for Rayleigh waves of the minimum group velocity at (15.6 sec) is 3.4 kms/sec. over this path we see that the velocity over the oceanic portion is 2.26 kms/sec. for the Dominican Republic Quake and 2.23 kms/sec. for the other. The velocity given by Press, Ewing & Jardetzky for this period is 1.6 kms/sec. and their minimum velocity occurs between periods 7-10 sec., for the model already described.

Comparison of our observations for Rayleigh waves in figs. (6) -- (7) with the theoretical curve of Jardetzky & Press (1953) seems to suggest that a decrease in ρ_2 to about 4.35 kms/sec. would probably give rise to better comparison between the two in the interval 40 - 70 sec. This decrease seems to be suggested by our observations for Love waves of quake (b) fig. (8), for which when extrapolated to higher periods a value of 4.35 kms/ sec. is indeed approached.

Consideration of the curves in fig. (8) for dispersing Love waves seems to indicate that the structure proposed by Evernden (1954) for Pacific crustal layering might possibly exist under the Atlantic. For it can be seen that our curve would fit quite well with curve B if the layer of ρ = 3.00 were to be increased to 11 or 12 kms and the shear velocity in the semi-infinite medium were decreased to 4.35 kms/sec. as has already been suggested from observed Rayleigh wave dispersion. It may also be thought curve "A" could be made to fit the observations if the layer's thickness were increased and ρ_2 were decreased, but the velocity for 13.0 sec. - 3.83 kms/sec. - and the general trend of our curve do not favor such a notion.

The above model of low velocity material 2.5 kms thick - consolidated sediment - overlying a layer of basalt of twice the thickness of that proposed by Jardetzky and Press may serve to explain our observations of Rayleigh waves in the 10 - 16 sec. period range. Comparison of quake (a) dispersion with the curve derived by Haskell (1951) for a two layered continental structure (fig.11) seems to point more and more in this direction.

This discussion of course does not rule out the Airy phase(corresponding to the minimum of the group velocity curve) since our observations have not been extended into the frequency range (7 - 10 sec. period) of this phenomenon. It does point out, however, that a layer capable of supporting shear overlying a layer of basalt should be seriously considered. We may also conclude from our comparison with Evernden's curve that Atlantic and Pacific layering is similar.

Recently Ewing and his co-workers (see Ewing, et al. (1950,'51, '54); Officer, et al. (1952); Hersey, et al. (1952), have established some interesting results in their seismic refraction studies in the Atlantic. A review of these papers will show discrepancies among the measurements of the investigators as regardslayering and velocities, however, they are not major ones. We will regard, then, Hersey's findings (1952) as representative of the results of their work, and quote them here for comparison with the conclusions obtained from the surface wave study.

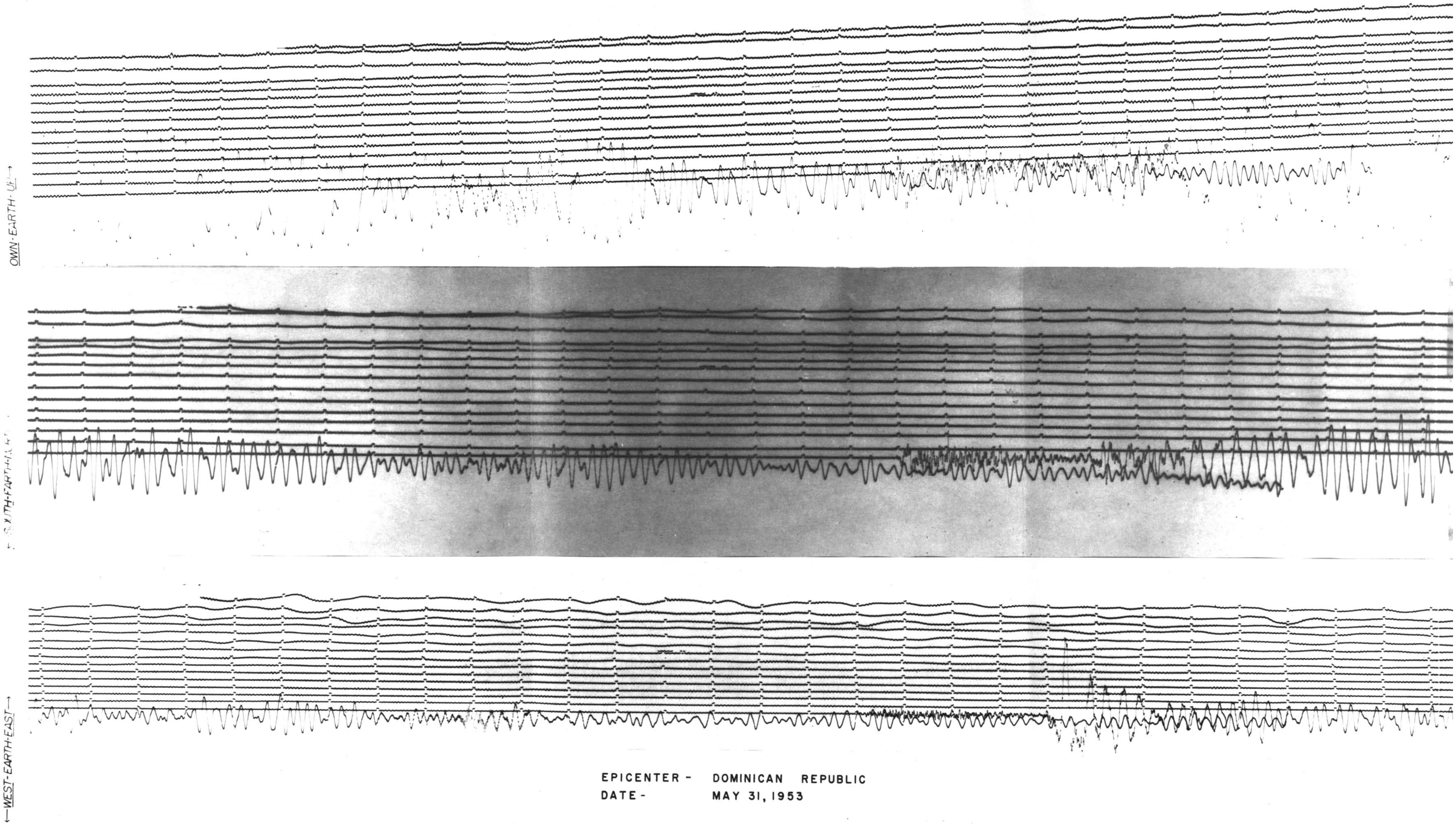
Hersey interprets his measurements in two ways, one of which is the following:

	Thickness		Velocity (Longitudinal Waves)
1..	5.30 kms	(Water)	1.51 kms./sec.
2..	0.42		1.69
3..	2.26		4.31
4..	2.49		6.64
5..			7.94

These results do point to the existence of low velocity material overlying a basaltic layer as was inferred from our comparison of the observed Love wave velocities with the curve calculated by Evernden. However, they do not bear out the 11 km. thickness of the basalt ($\alpha = 6.64$ km/sec.) which we surmised from the same curve (Ewing, et al. 1950, report a thickness of about 5 kms. and a velocity $\alpha =$ 6.42 km/sec.). It may well have been that if Evernden had regarded a still lower velocity overlying that which overlies the basalt such a thickness (11 kms.) would not have been required to obtain a decent fit with the observed values.

It is difficult, of course, to make statements concerning crustal layering from surface wave data because of the scarcity of theoretical curves which must be available for comparison. For instance, our observed Rayleigh wave dispersion may be accounted for by a structure consisting of three to five layers.

The improvements in observation of dispersion phenomena which we have presented certainly should warrant the calculation of such curves -- an undertaking which should not be too difficult with the means which have now been made available in the form of electronic computors.



EPICENTER - DOMINICAN REPUBLIC
DATE - MAY 31, 1953

PLATE I

TRAVELING SPECTRA — DOMINICAN REPUBLIC QUAKE

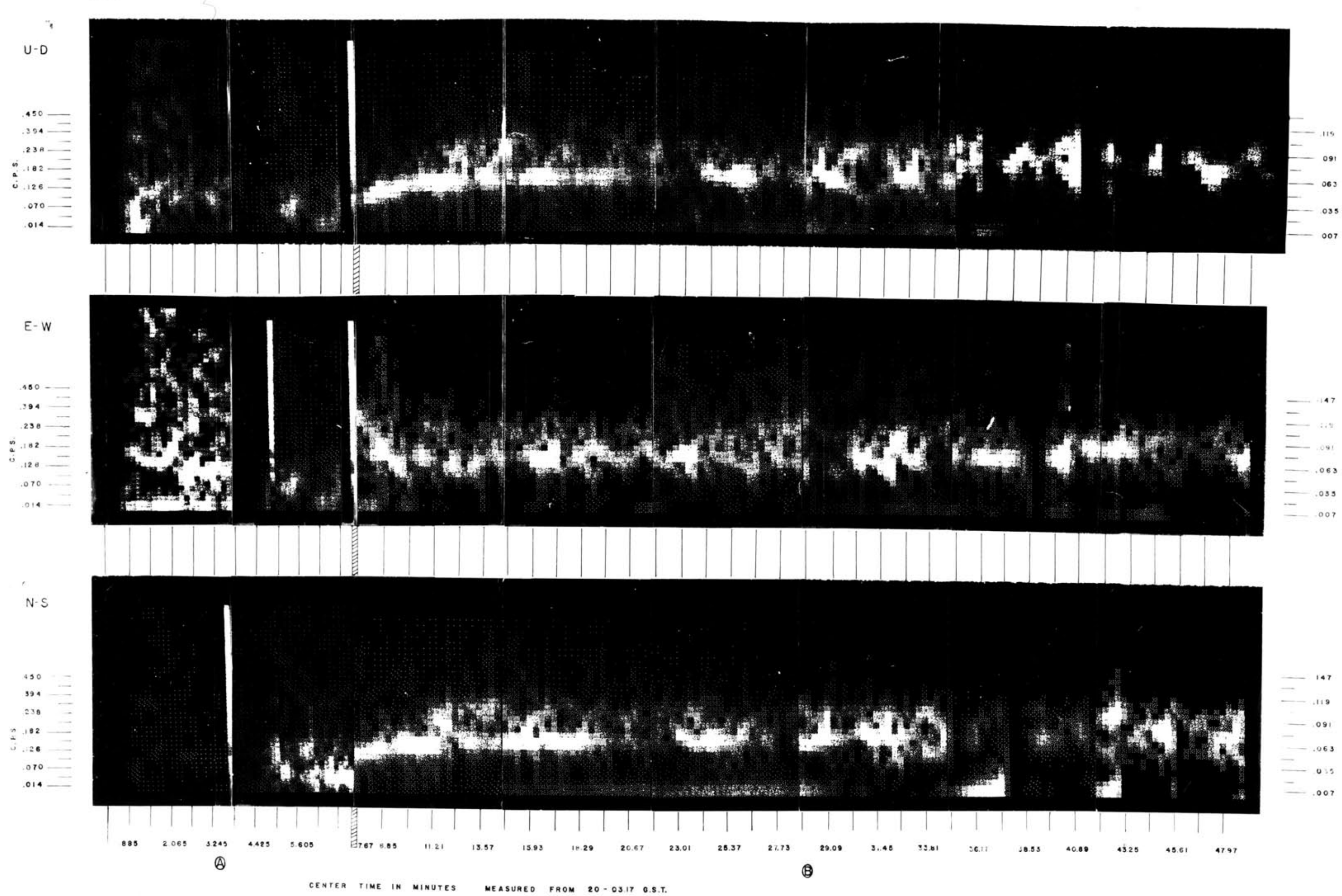
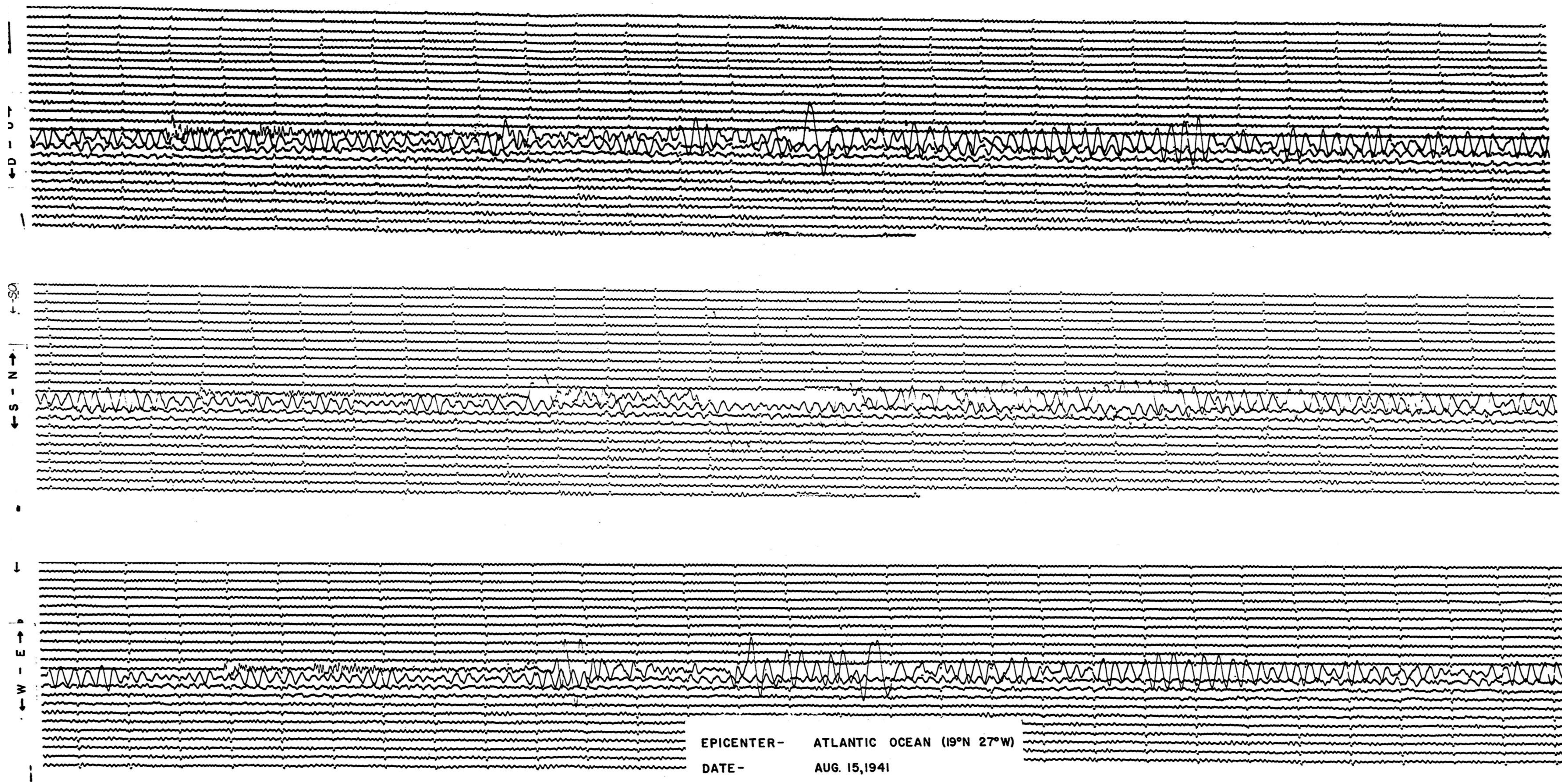


PLATE 2



EPICENTER- ATLANTIC OCEAN (19°N 27°W)
DATE- AUG. 15, 1941

PLATE 3

TRAVELING SPECTRA - ATLANTIC OCEAN QUAKE

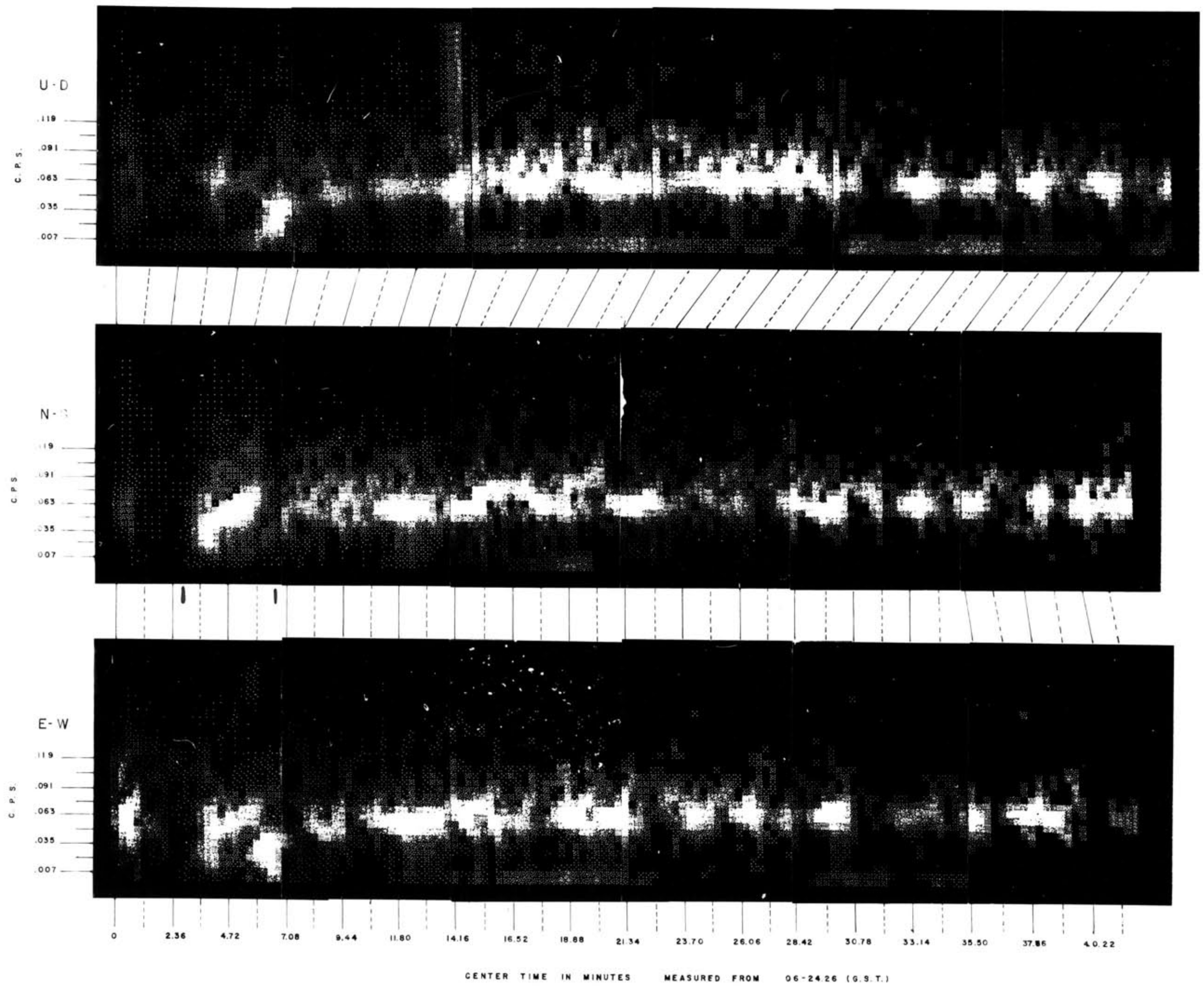


PLATE 4

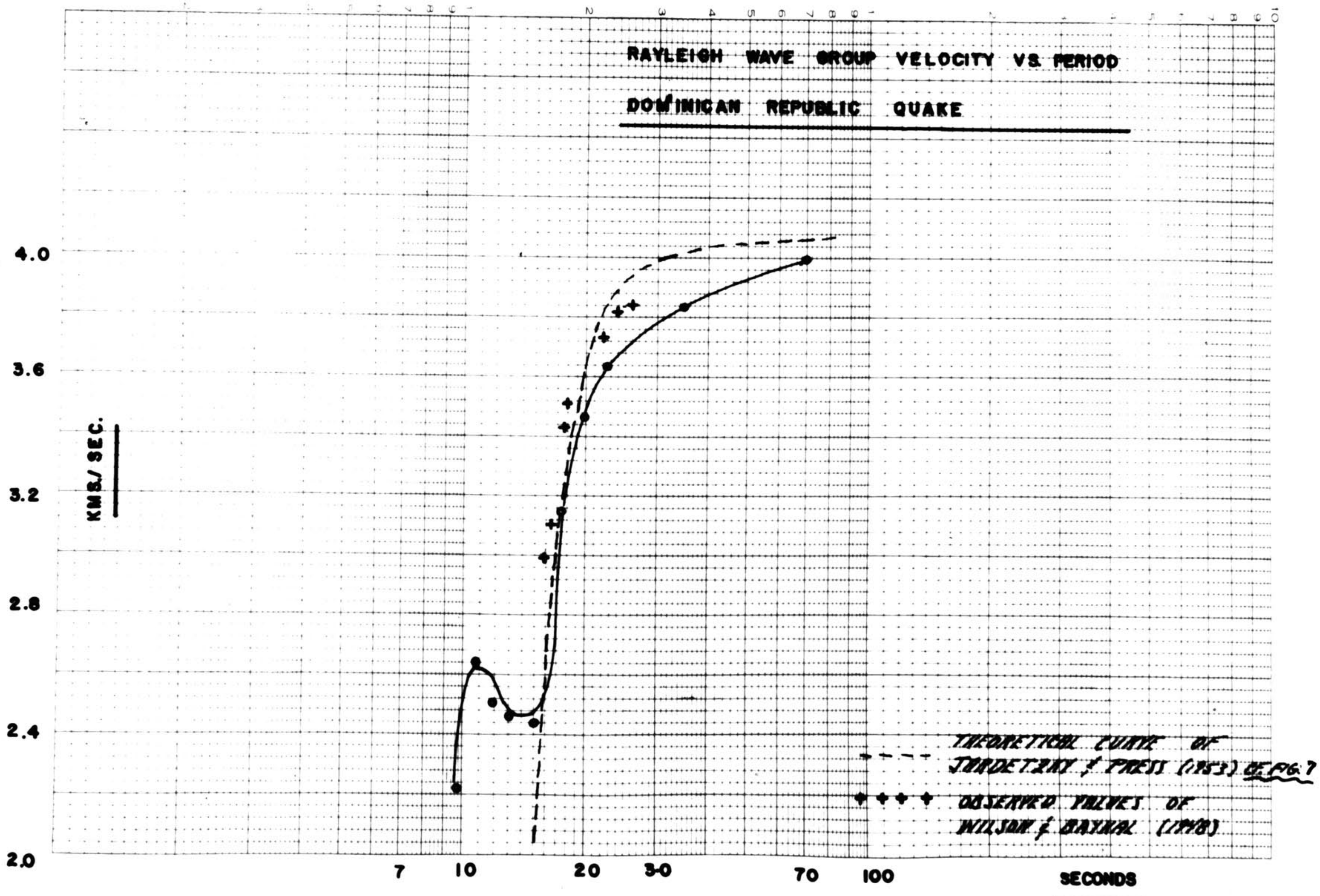
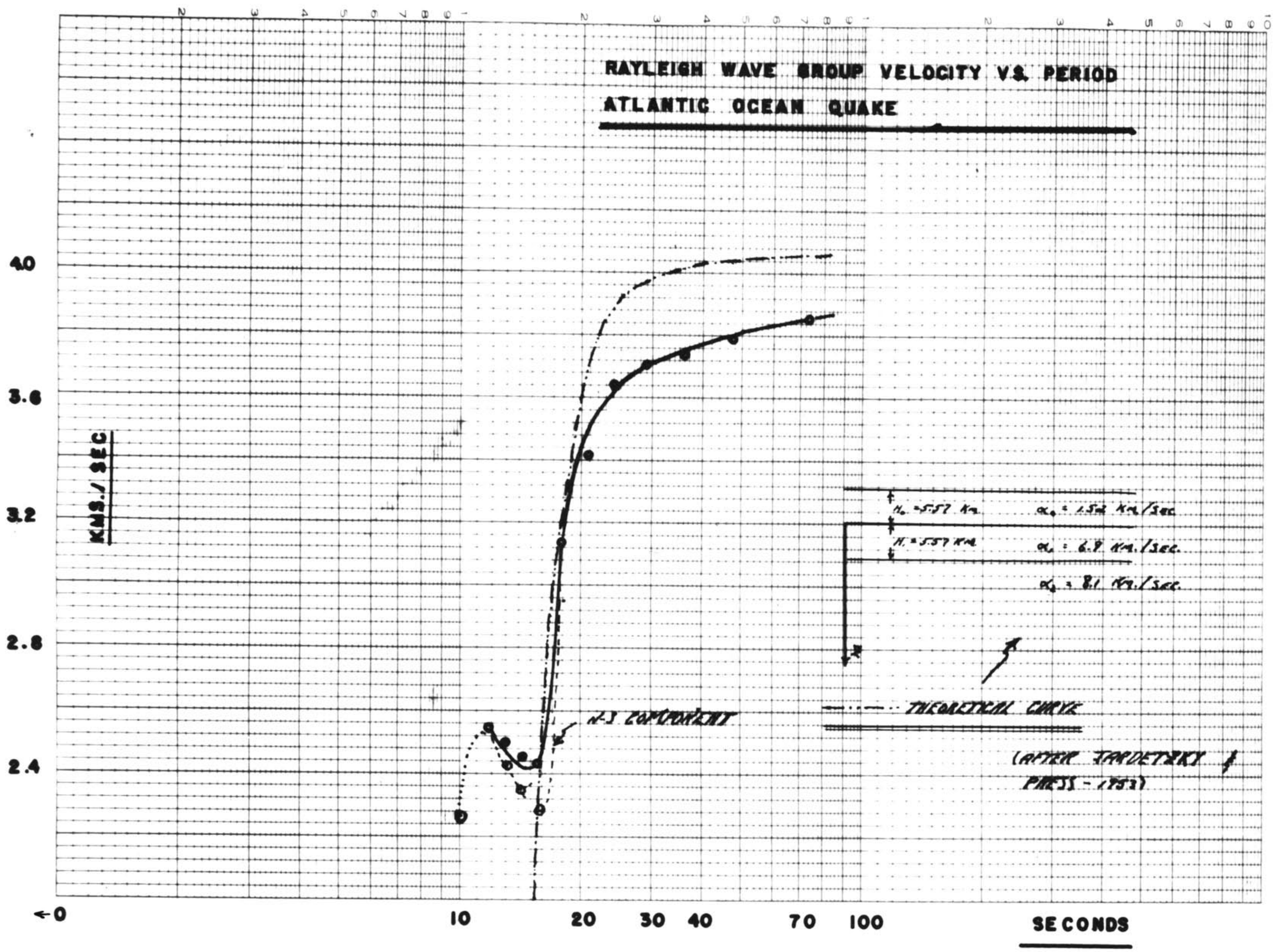
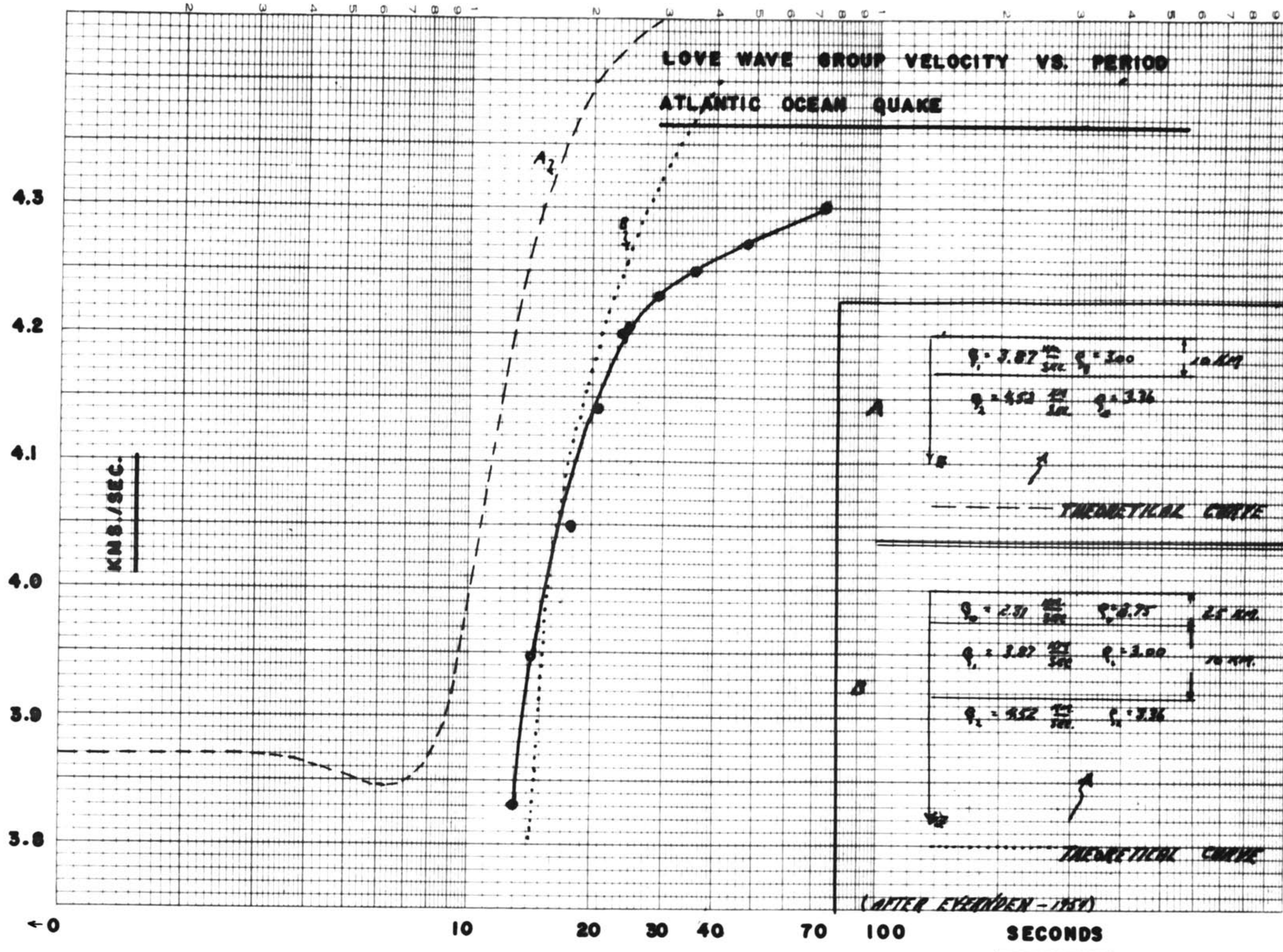

RAYLEIGH WAVE GROUP VELOCITY VS PERIOD
DOMINICAN REPUBLIC QUAKE
4.0
3.6
3.2
2.8
2.4
2.0
KMS/SEC.
7
10
20
3-0
70
100
SECONDS
THEORETICAL CURVE OF
JARDETZKY & PRESS (1953) OF FIG. 7
OBSERVED VALUES OF
WILSON & BAYKAL (1948)

Fig. (6)



RAYLEIGH WAVE GROUP VELOCITY V.S. PERIOD
ATLANTIC OCEAN QUAKE
KMS./SEC
4.0
3.6
3.2
2.8
2.4
←0
10
20
30
40
70
100
SECONDS
N-S COMPONENT
H_0 = 5.57 Km.
α_0 = 1.52 Km./Sec.
H_1 = 5.57 Km.
α_1 = 6.9 Km./Sec.
α_2 = 8.1 Km./Sec.
THEORETICAL CURVE
(AFTER JARDETZKY & PRESS - 1953)

Fig. (7)



LOVE WAVE GROUP VELOCITY VS. PERIOD
ATLANTIC OCEAN QUAKE
KMS./SEC.
4.3
4.2
4.1
4.0
3.9
3.8
A_2
A
B
THEORETICAL CURVE
THEORETICAL CURVE
(AFTER EVERNDEN - 1954)
← 0
10
20
30
40
70
100
SECONDS

Fig. (8)

TABLE I

Rayleigh Wave Group Velocity Dominican Republic Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC.
.014 c.p.s.	71.5 Sec	6.19 Min.	10.54 Min.	3.98
.028	35.7	6.53	10.88	3.84
.043	23.2	7.08	11.43	3.64
.049	20.4	7.67	12.02	3.45
.056	17.9	8.85	13.20	3.15
.063	15.6	12.98	17.33	2.41
.077	13.0	12.68	17.03	2.44
.084	11.9	12.39	16.74	2.48
.091	11.0	11.50	15.85	2.62
.098	10.1	14.46	18.81	2.22

TABLE II

Rayleigh Wave Group Velocity Atlantic Ocean Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC.
.014 c.p.s.	71.5 Sec.	5.90 Min.	21.16 Min.	3.83
.021	47.7	6.05	21.31	3.79
.028	35.7	6.19	21.45	3.76
.035	28.6	6.49	21.75	3.72
.042	23.8	7.08	22.34	3.63
.049	20.4	8.45	23.71	3.41
.056	17.9	10.62	25.88	3.12
.063	15.6	18.29	33.55	2.42
.070	14.3	17.70	32.96	2.46
.077	13.0	17.11	32.37	2.50
.084	11.9	16.81	32.07	2.53
# .098 N-S	10.1	20.06	35.32	2.27

Questionable

TABLE III

Love Wave Group Velocity Atlantic Ocean Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC.
.014 c.p.s.	71.5 sec.	3.54 Min.	18.80 Min.	4.30
.021	47.7	3.68	18.94	4.27
.028	35.7	3.77	19.03	4.25
.035	28.6	3.83	19.09	4.23
.042	23.8	3.97	19.23	4.20
.049	20.4	4.26	19.52	4.14
.056	17.9	4.72	19.98	4.05
.070	14.3	5.31	20.57	3.95
.077	13.0	5.90	21.16	3.83

b. Case II — Continental Path

(1) Data Analyzed --- Methods Employed

The data listed below was obtained from the U.S. Coast and Geodetic Survey.

Quake (c)	Date	- Oct. 13, 1953
	Epicenter	- N. Gulf Calif. 30 N $113\frac{1}{2}$W
	Time	- 08:53:45 G.S.T.
	Magnitude	- $6\frac{1}{2}$ (Pas.)
	Epicentral Distance	- 3950 kms.
Quake (d)	Date	- Dec. 4, 1953
	Epicenter	- Coast Vancouver Is. $49\frac{1}{2}$ N 129 W
	Time	- 14:54:46 G.S.T.
	Magnitude	- $6\frac{1}{2}$ (Pas.)
	Epicentral Distance	- 4450 kms.

Photostatic reproductions of the records of these quakes may be found on plates (5) and (7).

Data points were read from all seismograms at approximately every .894 seconds; 40 point intervals were utilized in all spectra calculations. As a result of this choice n-harmonic frequencies of .014 C.P.S. = o, 1,2. ... 40 were calculated for each interval.

Density plots of the traveling spectra computed for quake (c) and (d) are displayed in plates (6) and (8) respectively.

The methods of obtaining the group velocity curves have already been mentioned under Case I. Our observations are tabulated in Tables IV - VII and plotted in figures (9) - (12). Since there was some question in our mind as to how a smooth curve could be drawn through the density plot (U D component) of the Vancouver Is. quake we have plotted as a result three curves for the Rayleigh wave group velocity. Of these three, curve "B" seems to us to be the most plausible, after consideration of travel time and velocity plots. Curve "A" was drawn in its present form to display possible comparison between the two continental paths analyzed, although it could have been extended as the two plotted points indicate. However, the fact that the excitation of more than one mode might have occurred to give rise to the two curves should not be overlooked.

We have also reproduced for purposes of comparison the theoretical curves for continental Rayleigh curve dispersion derived by Haskell (1951) on fig. (11). Figure (9) contains the curve derived by Wilson and Baykal (1948). There is also included on this same figure the re cent observations of Brilliant and Ewing (1954) of Rayleigh wave dispersion across the United States. We have also computed several Love wave group velocity curves of assumed continental structure and have plotted same on figures (10) and (12). The assumptions pertinent to each theoretical curve are contained on the respective graphical plot.

(2) Discussion

Consideration of our dispersion curves for both Love and Rayleigh waves show marked dissimilarity. Generally speaking, however, they clearly show that corresponding groups exhibit higher velocities over Canada and Northern United States than they do over Southern portions of the United States. This fact is probably attributable to greater thicknesses of sialic rock over the latter route.

The observations of Briliant and Ewing (1954) of Rayleigh wave dispersion across the United States were made from quakes (Pacific) which took the <u>same</u> continental path to Weston as did our N. Gulf California quake. It is interesting to note the similarity between our values and theirs.

Only in one case, namely, the Vancouver Is. quake, is it possible to envision comparison between a theoretical curve and the observed values. In the curves, fig. (11) for Rayleigh wave velocities we do see some similarity between curve B and curve I -- that which was derived by Haskell (1951) from a two layer case where velocity increases with depth. Due to the complexity of the calculations it is difficult to say in what manner the various parameters involved should be changed to bring about a decent fit. We will, however, venture to speculate that an increase in ρ_1, and ρ_2, a decrease in ρ_3, an increase in the first layers thickness, and a decrease in the thickness of the seond layer might serve to accomplish the fit between the two curves.

There seems to be no doubt that the scatter of values obtained for the two paths was effected by a high degree of heterogeneity in the upper crustal layers. Such a fact of necessity rules out the method of using a number of quakes to determine a general continental dispersion curve. It appears then that any study of continental surface wave dispersion must take into consideration direction of approach to the station and the epicentral distance, because various paths will most likely entail different layering.

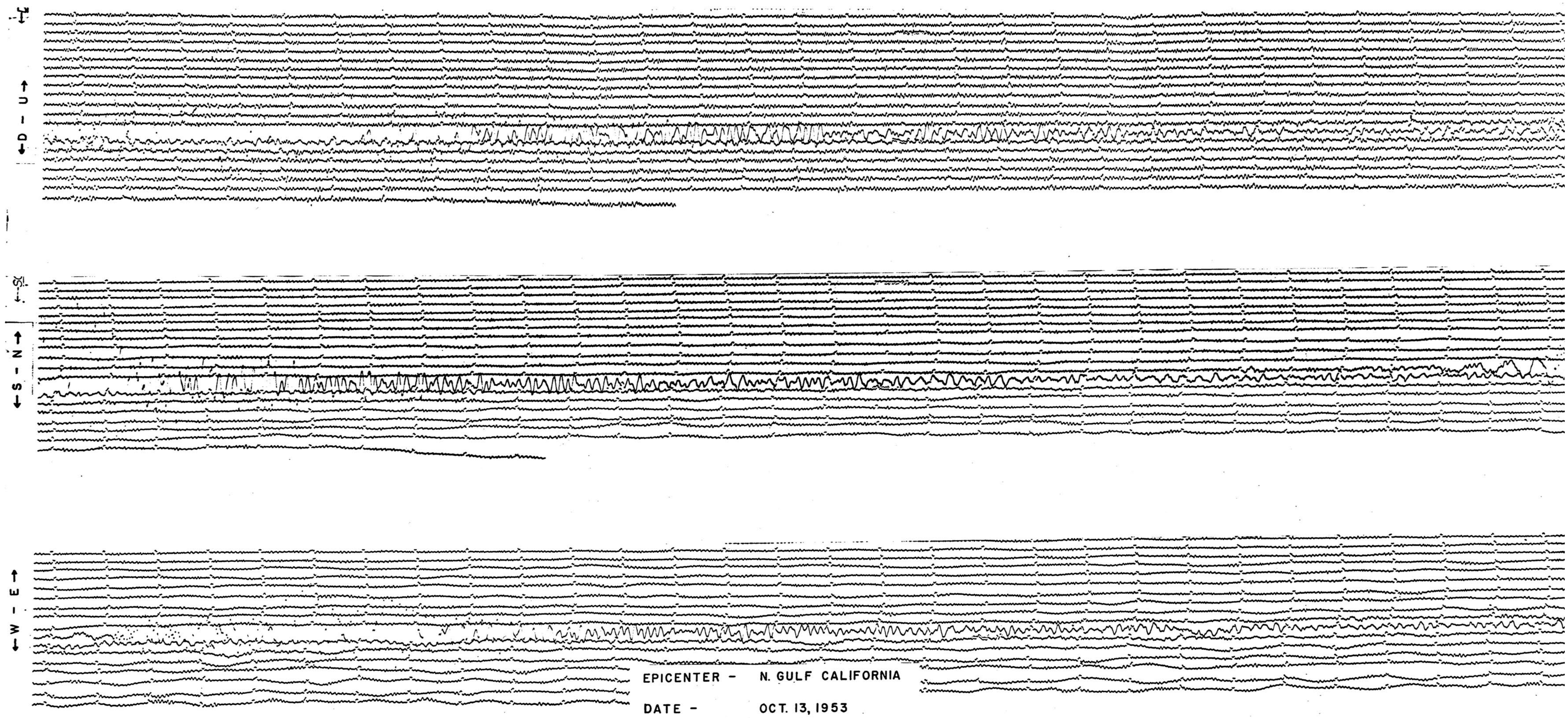


PLATE 5

TRAVELING SPECTRA — N. GULF CALIFORNIA QUAKE

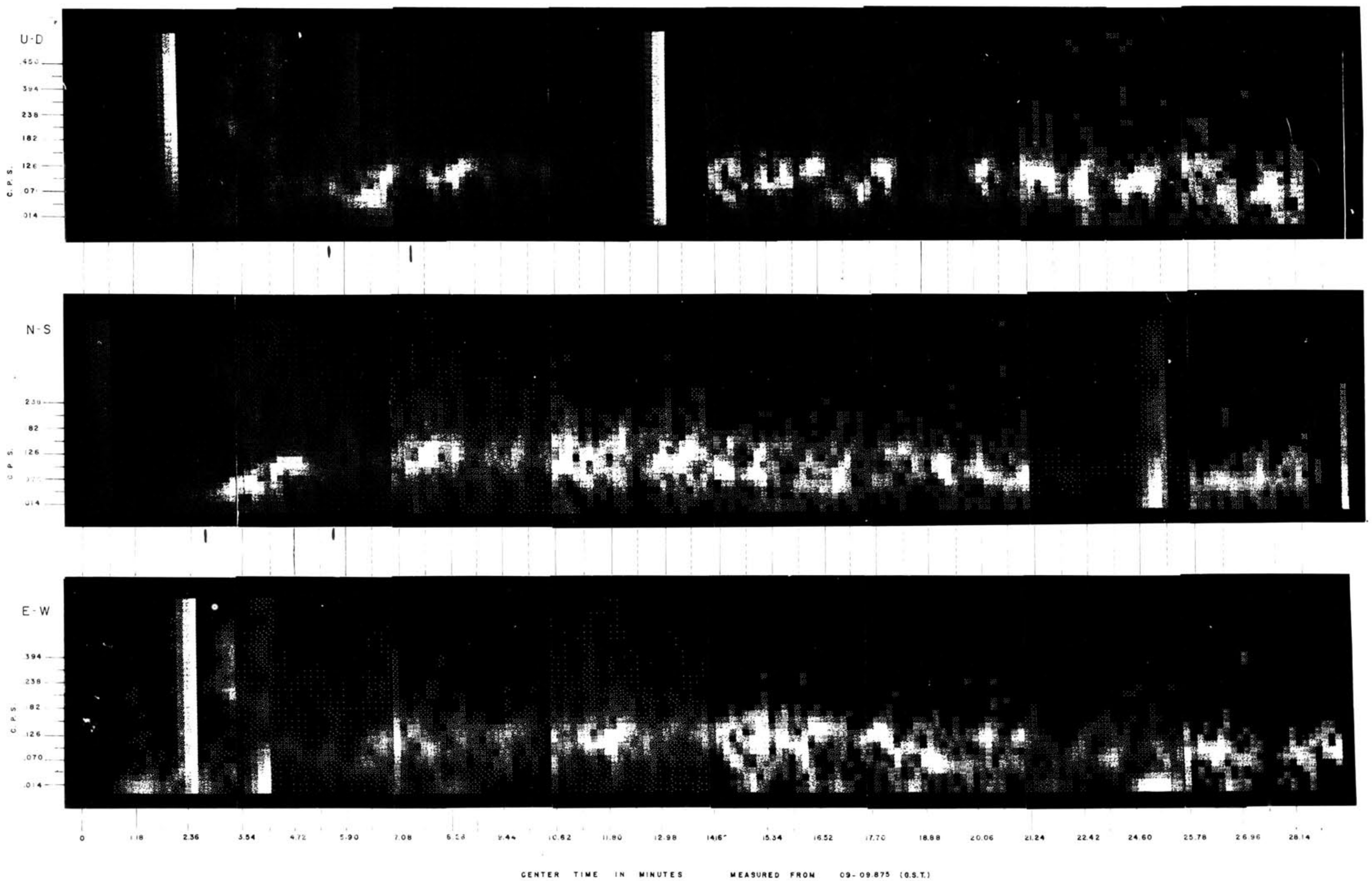
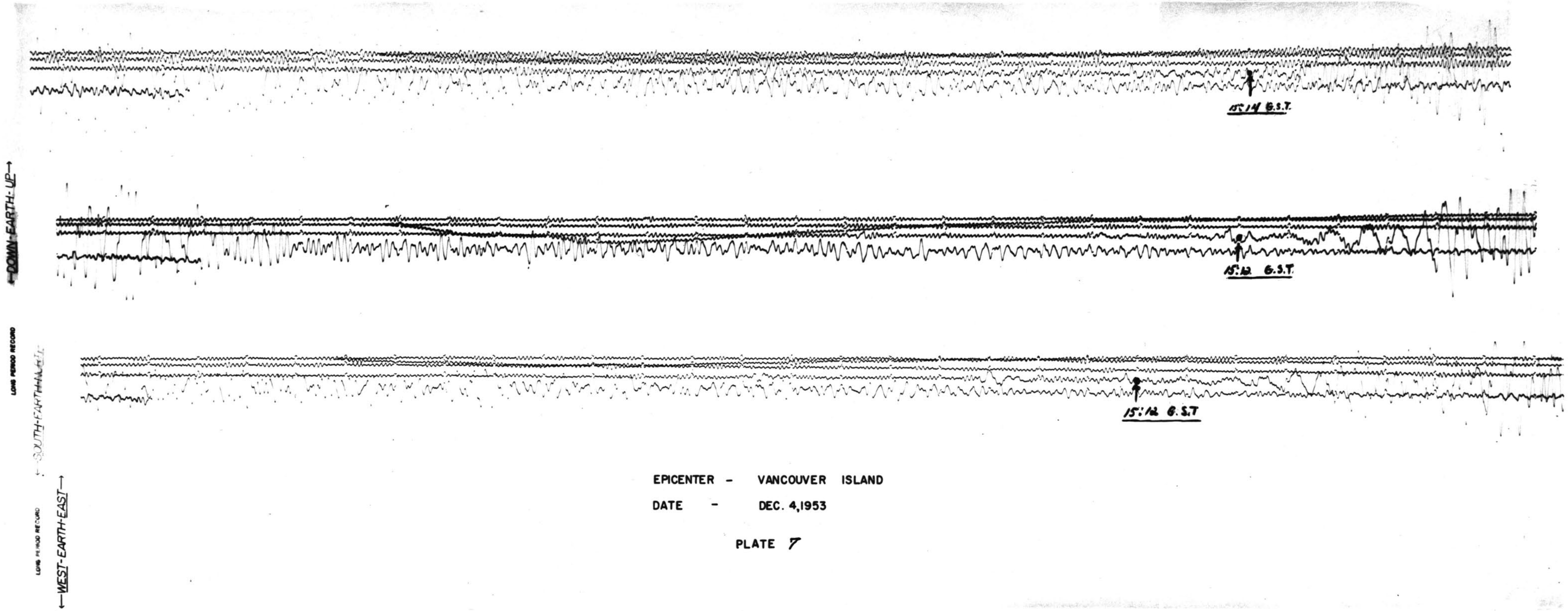


PLATE 6



EPICENTER - VANCOUVER ISLAND

DATE - DEC. 4,1953

PLATE 7

TRAVELING SPECTRA - VANCOUVER IS. QUAKE

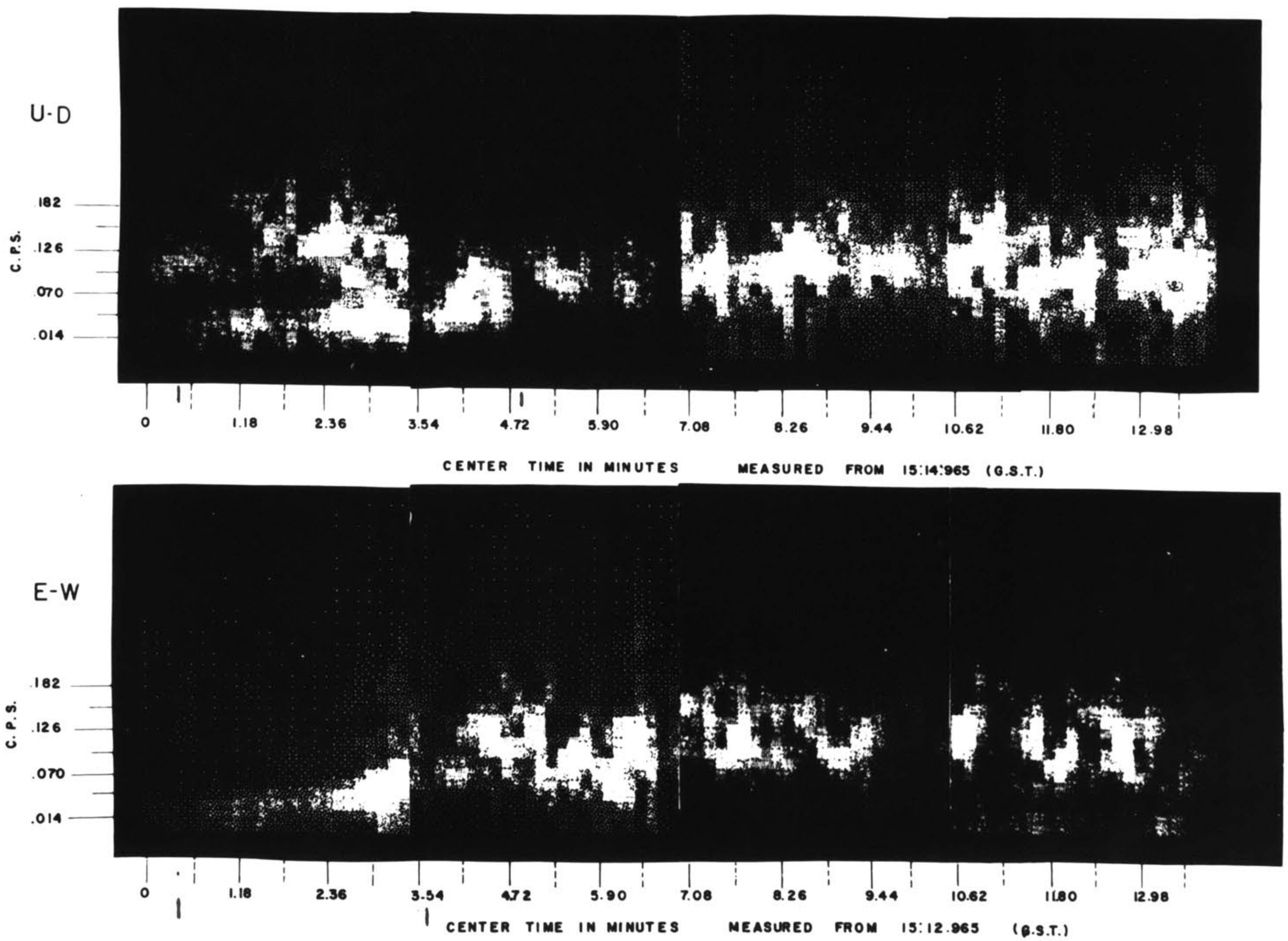
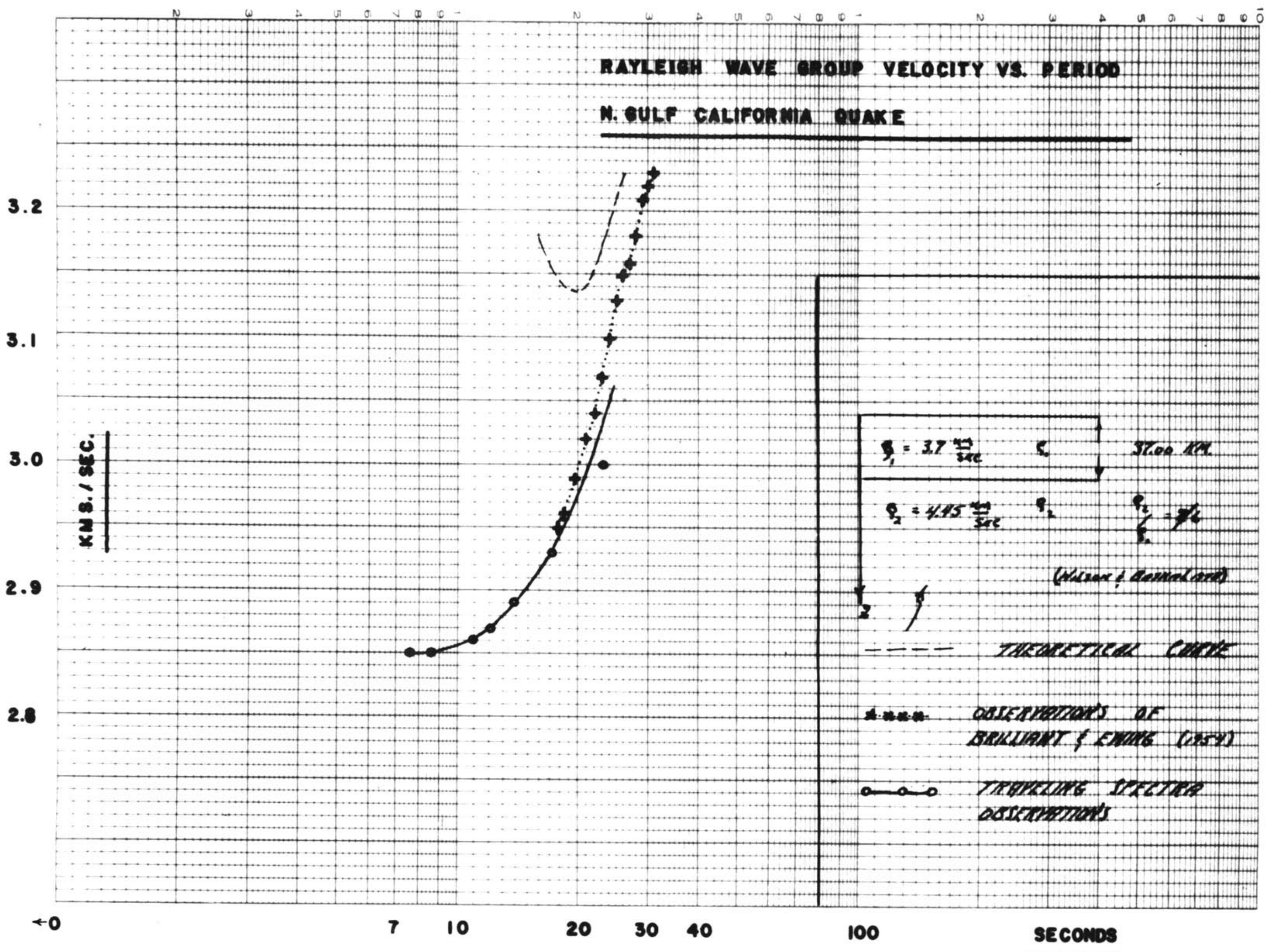


PLATE 8



RAYLEIGH WAVE GROUP VELOCITY VS. PERIOD
N. GULF CALIFORNIA QUAKE
KMS./SEC.
3.2
3.1
3.0
2.9
2.8
0
7
10
20
30
40
100
SECONDS
THEORETICAL CURVE
OBSERVATIONS OF BRILLIANT & EWING (1954)
TRAVELLING SPECTRA OBSERVATIONS

Fig. (9)

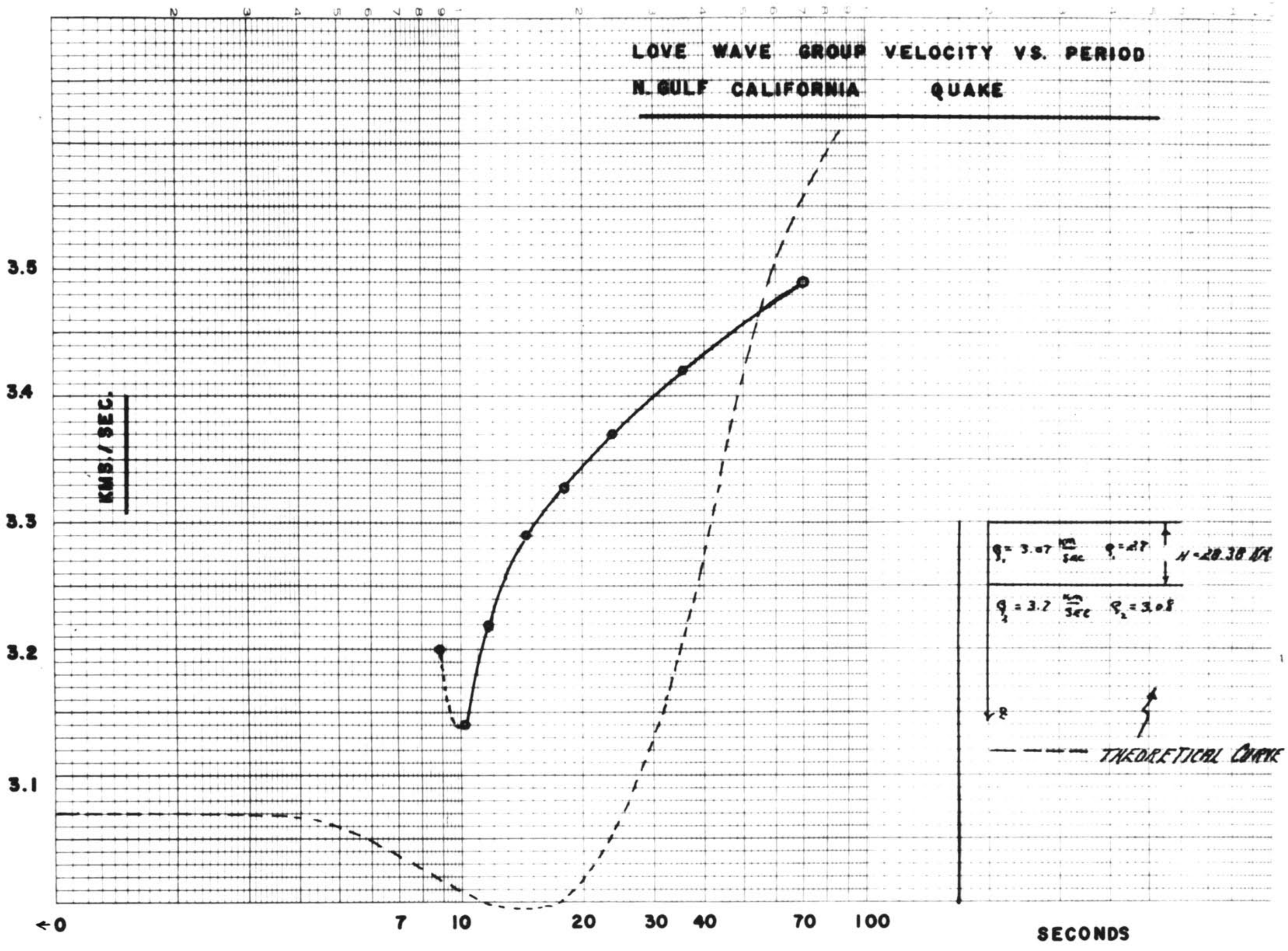
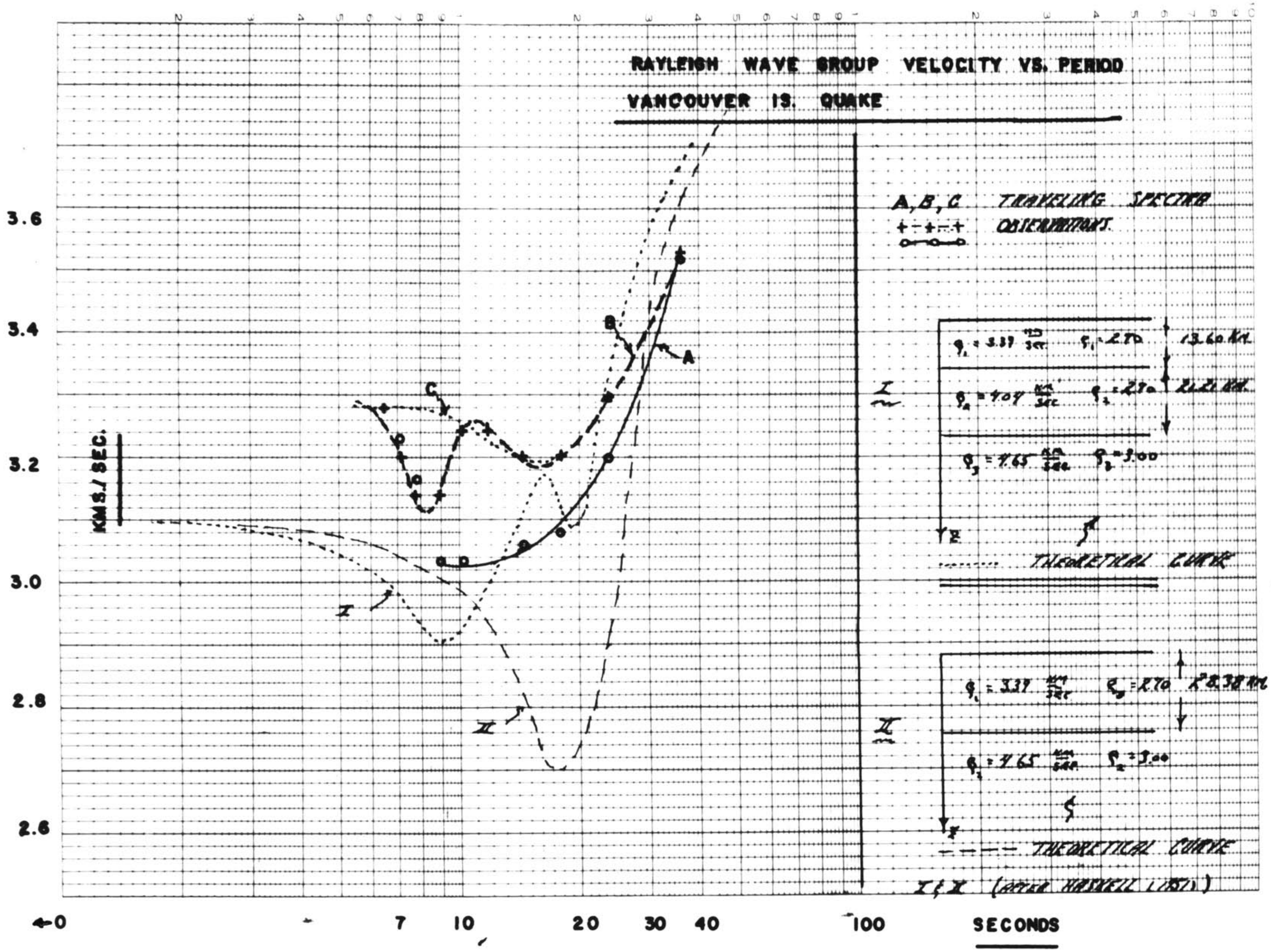

LOVE WAVE GROUP VELOCITY VS. PERIOD
N. GULF CALIFORNIA QUAKE
KMS./SEC.
3.5
3.4
3.3
3.2
3.1
← 0
7
10
20
30
40
70
100
SECONDS
H = 28.38 KM.
THEORETICAL CURVE

Fig. (10)



RAYLEIGH WAVE GROUP VELOCITY VS. PERIOD
VANCOUVER IS. QUAKE
KMS./SEC.
3.6
3.4
3.2
3.0
2.8
2.6
7
10
20
30
40
100
SECONDS
A
B
C
A, B, C TRAVELING SPECTRA
OBSERVATIONS
THEORETICAL CURVE
THEORETICAL CURVE
I & II (AFTER HASKELL (1951))

Fig. (11)

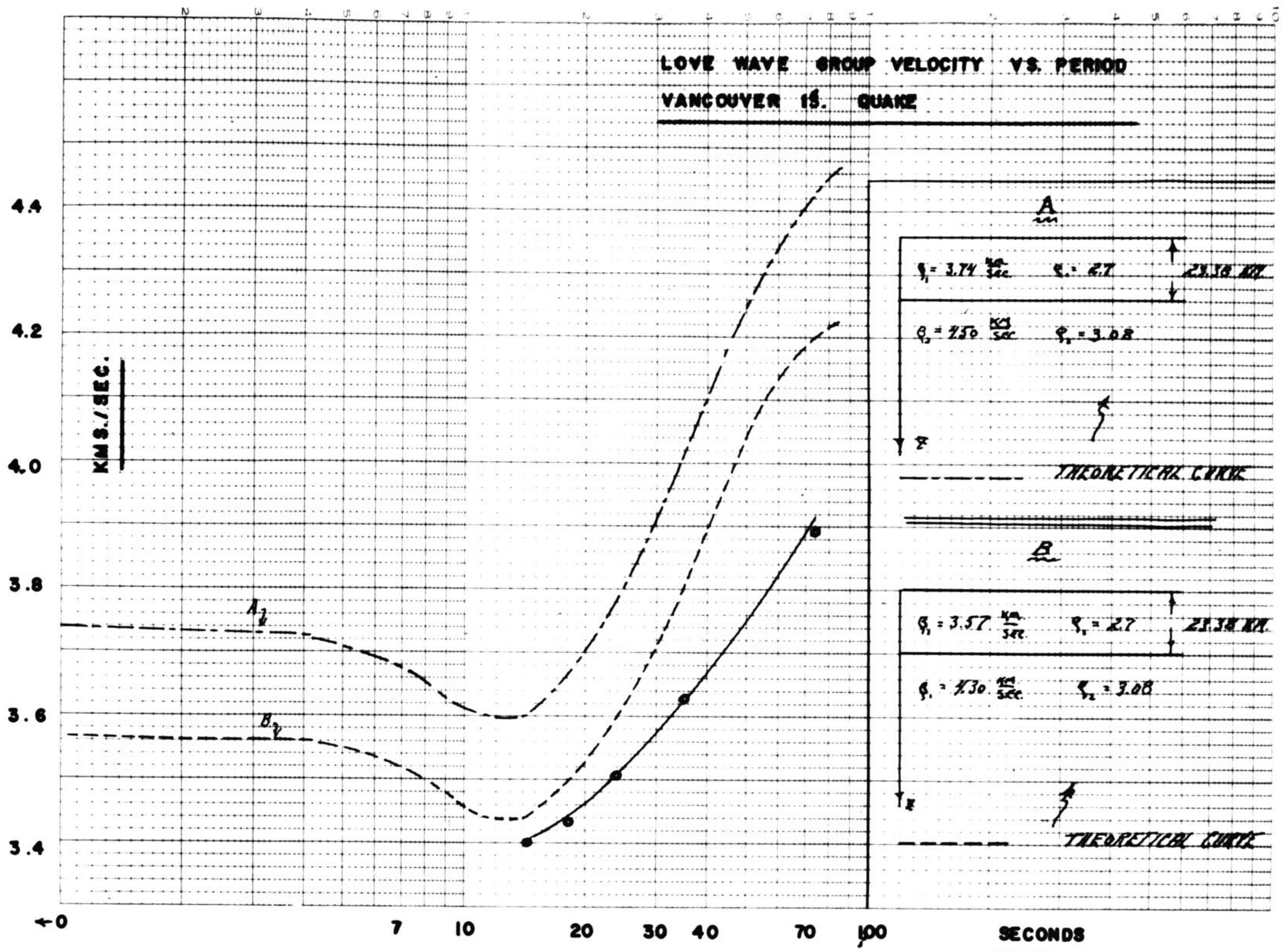

LOVE WAVE GROUP VELOCITY VS. PERIOD
VANCOUVER IS. QUAKE
KMS./SEC.
4.4
4.2
4.0
3.8
3.6
3.4
A_2
B_2
←0
7
10
20
30
40
70
100
SECONDS
A
$\beta_1 = 3.74$ km/sec
$\rho_1 = 2.7$
23.38 KM
$\beta_2 = 4.50$ km/sec
$\rho_2 = 3.08$
THEORETICAL CURVE
B
$\beta_1 = 3.57$ km/sec
$\rho_1 = 2.7$
23.38 KM
$\beta_2 = 4.30$ km/sec
$\rho_2 = 3.08$
THEORETICAL CURVE

Fig. (12)

TABLE IV

Rayleigh Wave Group Velocity N. Gulf California Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
.028 c.p.s.	35.7 sec.	Min.	Min.	
.042	23.8	6.04	22.125	2.99
.056	17.9	6.37	22.495	2.93
.070	14.3	6.66	22.785	2.89
.084	11.9	6.80	22.925	2.87
.098	10.1	6.87	22.990	2.86
.112	8.9	6.93	23.055	2.85
.126	7.95	6.93	23.055	2.85

TABLR V

Love Wave Group Velocity N. Gulf California Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
.014 c.p.s.	71.5 sec.	-- Min.	-- Min.	
.028	35.7	3.097	19.222	3.42
.042	23.8	3.393	19.518	3.37
.056	17.9	3.54	19.665	3.35
.070	14.3	3.834	19.959	3.29
.084	11.9	4.277	20.402	3.22
.098	10.1	4.86	20.985	3.14
# .112	8.9	4.426	20.846	3.20

Uncertain

TABLE VI

Rayleigh Wave — Group Velocity — Vancouver Is. Quake

(Curve A)

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
.028 c.p.s.	35.7 Sec.	1.47 Min.	21.66 Min.	3.52
.042	23.8	2.95	23.14	3.2
.056	17.9	3.99	24.19	3.08
.070	14.3	4.13	24.32	3.06
.098	10.1	4.72	24.46	3.04
.112	8.9	4.72	24.46	3.04
.126	7.95	3.25	23.44	3.17
.140	7.14	2.36	22.55	3.28

(Curve B)

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
,028	35.7	1.47	21.66	3.52
.042	23.8	2.22	22.41	3.31
.056	17.9	2.95	22.14	3.21
.070	14.3	2.95	23.14	3.21
.082	11.9	2.65	22.84	3.25
.098	10.1	2.65	22.84	3.25
.112	8.9	3.25	23.44	3.15
.126	7.9	3.25	23.44	3.15
.140	7.1	2.94	23.13	3.20
.154	6.5	2.36	22.55	3.28

TABLE VII

Love Wave — Group Velocity — Vancouver Is. Quake

FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
.014 c.p.s.	71.4 Sec.	.89 Min.	19.37 Min	3.84
.028	35.7	2.22	20.41	3.63
.042	23.8	2.95	21.14	3.51
.056	17.9	3.25	21.60	3.44
.070	14.3	3.39	21.72	3.40

(C) Case III -- Pacific and Continental Path

(1) Data Analyzed -- Methods Employed

The data given below was obtained from the United States Coast and Geodetic Survey.

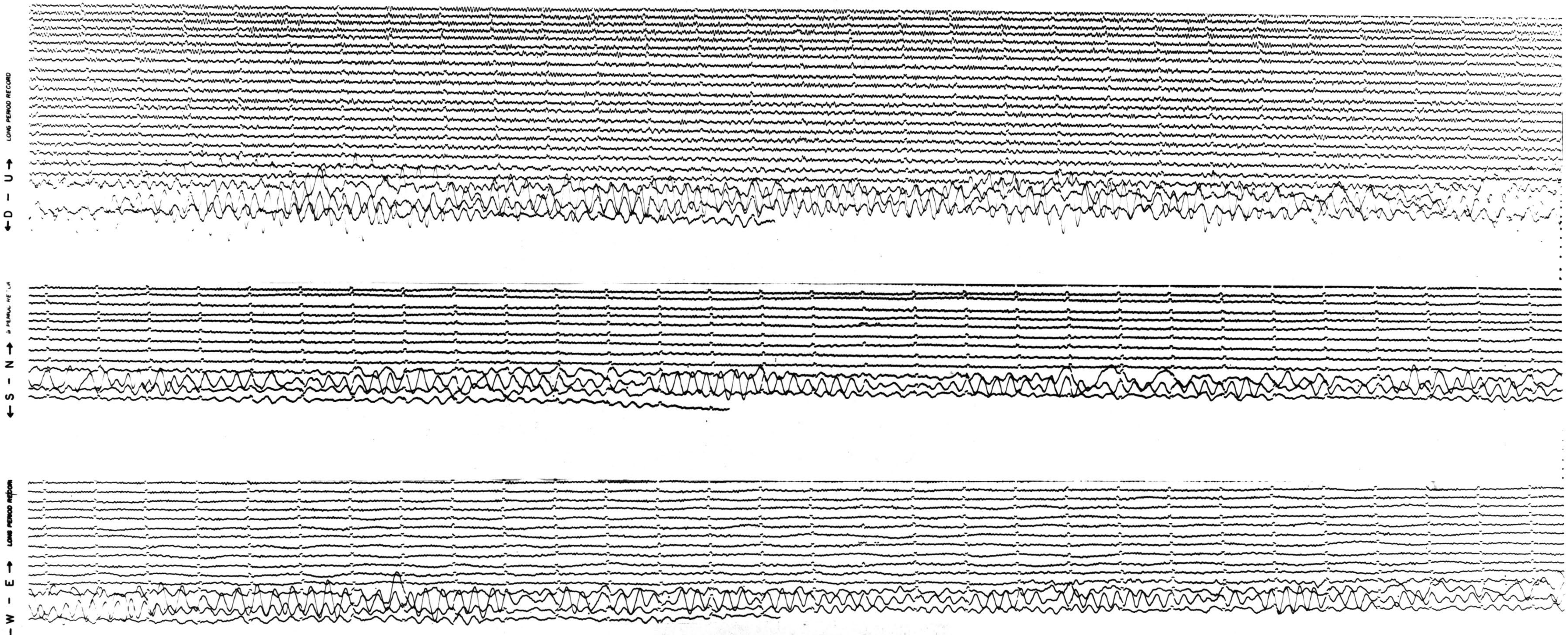
Quake (e) Date - February 26, 1953
Epicenter - Santa Cruz Isles 11° S 164½E
Time - - 11:42:26 G.S.T.
Magnitude - 7¼ (Pas.)
Epicentral Distance - 13,650 kms.

Photostatic reproduction of the component seismogrzms of this quake may be found on plate (9).

Readings were taken from the record every 1.78 sec., and n-harmonic frequencies of .007 c.p.s. (n = 0,1,2,--40) were computed from intervals consisting of 40 points. The density plot of the calculated traveling spectra are photographed on plate (10).

The same procedure as mentioned previously in drawing our dispersion curves was employed here in plotting the Rayleigh wave group velocity curve. Since there was some evidence of generation of a shearing motion transverse to the direction of propagation (Love waves) we corrected our observed values of Rayleigh and Love motion for continental dispersion effects, in order to ascertain which groups -- if any -- were generated at the western coast of the continent - (Our findings in this regard will be presented in a following section). The curves used for continental correction were our Rayleigh (curve B) and Love wave observations for the Vancouver Is. quake, since the continental path of the two were practivally the same. The groups which were found to be generated at the source were utilized in plotting our Love wave curve for the continental path - Fig. (14).

For comparison purposes we have subtracted the continental Rayleigh wave dispersion (curve B, Fig. (11) from that of the combin ed path, and have plotted the results in Fig. (13). We assumed that the continental path was approximately 4650 kms (the distance from the 2000 fatham line off Vancouver to Weston, Mass.). A similar curve was plotted for the Love waves on Fig. (14).



EPICENTER - SANTA CRUZ ISLES

DATE - FEB. 26, 1953

PLATE 9

TRAVELING SPECTRA — SANTA CRUZ ISLES QUAKE

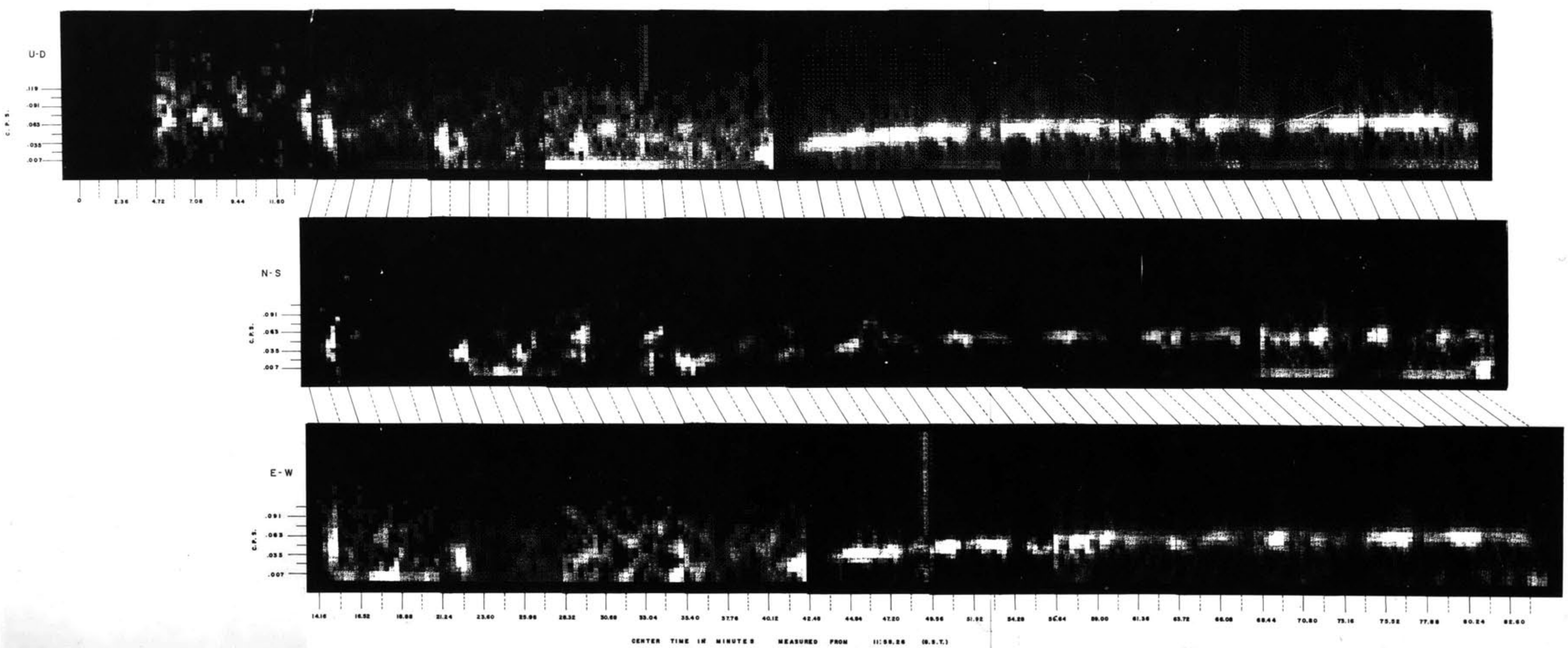


PLATE 10

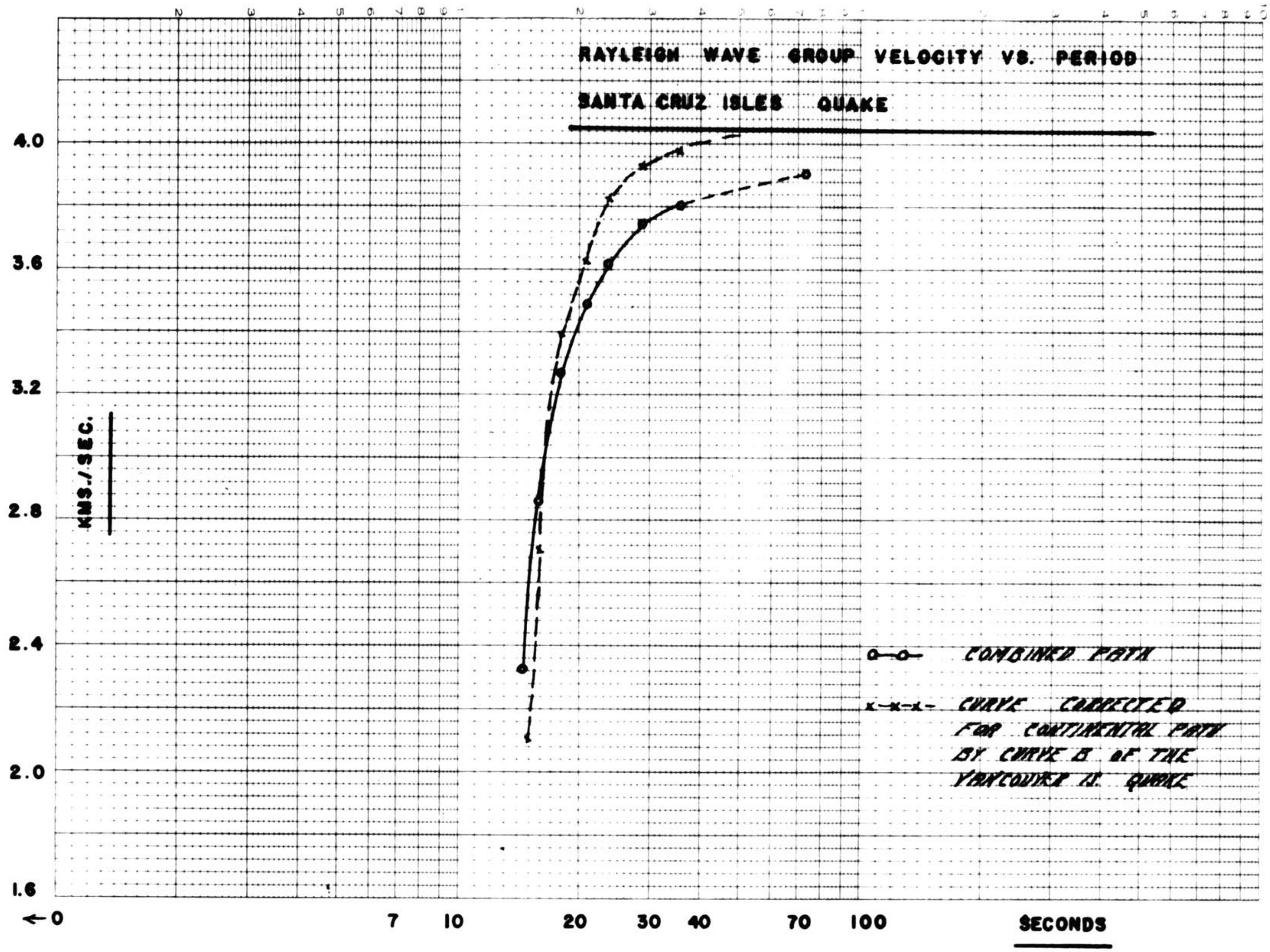
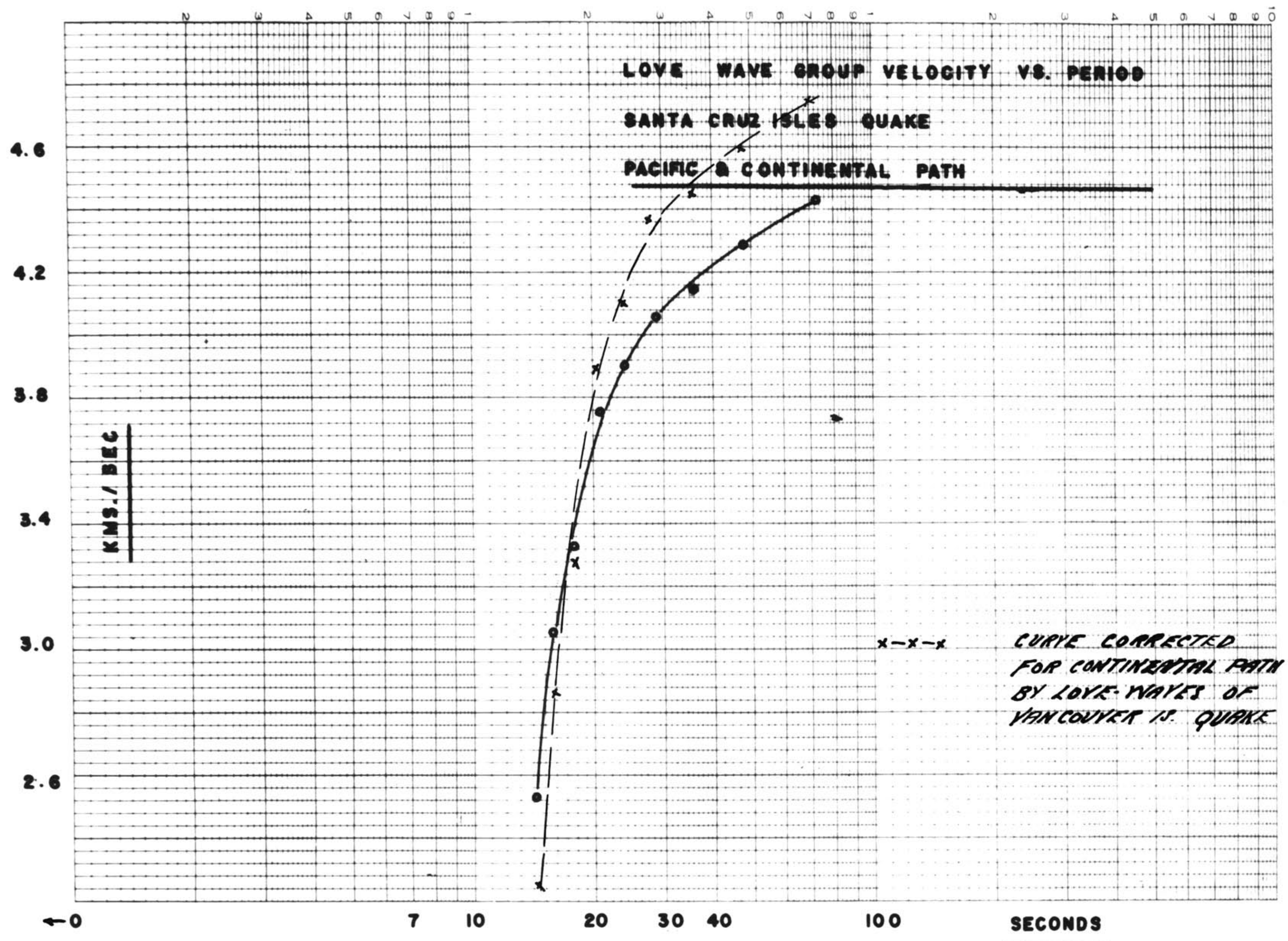

RAYLEIGH WAVE GROUP VELOCITY VS. PERIOD
SANTA CRUZ ISLES QUAKE
4.0
3.6
3.2
2.8
2.4
2.0
1.6
KMS./SEC.
0
7
10
20
30
40
70
100
SECONDS
COMBINED PATH
CURVE CORRECTED FOR CONTINENTAL PATH BY CURVE B OF THE VANCOUVER IS. QUAKE

Fig. (13)

Fig. (14)



LOVE WAVE GROUP VELOCITY VS. PERIOD
SANTA CRUZ ISLES QUAKE
PACIFIC & CONTINENTAL PATH
KMS./SEC
4.6
4.2
3.8
3.4
3.0
2.6
0
7
10
20
30
40
100
SECONDS
x—x—x
CURVE CORRECTED FOR CONTINENTAL PATH BY LOVE-WAVES OF VANCOUVER IS. QUAKE

76

TABLE VIII

Rayleigh Wave		Group Velocity	Santa Cruz Isles Quake	
FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
# .014 c.p.s.	71.5 Sec.	42.480 Min.	58.310 Min.	3.90
.028	35.7	43.955	59.785	3.80
.035	28.6	45.135	60.965	3.74
.042	23.8		62.735	3.62
.049	20.4	49.560	65.390	3.49
.056	17.9	53.690	69.520	3.27
.063	15.9	63.72	79.550	2.86
.020	14.3	61.42	97.25	2.33

Questionable

TABLE IX

Love Wave		Group Velocity	Santa Cruz Isles Quake	
FREQUENCY	PERIOD	CENTER TIME	TRAVEL TIME	VELOCITY KMS/SEC
.014 c.p.s.	71.5 Sec.	35.99 Min.	51.82 Min.	4.42
.021	47.6	37.17	53.10	4.29
.028	35.7	38.94	54.77	4.15
#.035	28.6	40.12	55.95	4.06
#.042	23.8	42.48	58.33	3.91
#.049	20.4	44.84	60.67	3.75
.056	17.9	52.51	68.34	3.31
.063	15.9	59.00	74.83	3.04
.070	14.3	73.75	89.58	2.55

Questionable

C... A Study of Reflected, Refracted and Direct Body Waves.

(a) The Reflected and Refracted Phases.

In the foregoing sections it was noted on those seismograms which were analyzed over times prior to the arrival of the dispersing surface wave groups, that the traveling spectra gave evidence of what appeared to be pulses of energy arriving at discrete intervals of time. Suspecting that these occurrences might well be correlated with the times of arrival of the various P and S phases we undertook to compare the center times of these pulses with the travel times given by Gutenberg and Richter (1939) and now circulated by the United States Coast & Geodetic Survey. The results were very encouraging, displaying in each instance remarkable correlation in arrival time.

We have arbitrarily chosen our maximum error as being the time from the center-time of the interval to the extremities of the interval in question.

Hence we have for

Dominican Republic Quake --	±.295 Min.	--Region	A
	±.590 "	-- "	B
N. Gulf California Quake --	±.295 "		
Atlantic Ocean Quake --	±.295 "		
Santa Cruz Isles Quake -	±.590 "		

In Tables X - XIII below we have listed the results of this investigation for four of the quakes analyzed in this report.

TABLE X

Dominican Republic Quake

Phase	Dominant Frequencies	Travel Time (Gutenberg)	Travel Time (Trav.Spec.)	Center Time
P	.000-.070 c.p.s.	5.00 Min.	5.23 Min.	.885 Min.
PP	.000 - .014	5.50	5.28	1.63
S	.070 - .560	8.70	8.18	3.83
Love	.126 - .560	9.15	9.22	4.87

TABLE XI

Atlantic Ocean Quake

Phase	Dominant Frequencies	Travel Time (Gutenberg)	Travel Time (Trav.Spec.)	Center Time
S H	.075 - .063 c.p.s.	14.90 Min.	15.85 Min.	.59 Min (E-W)

TABLE XII

N.Gulf California Quake

Phase	Dominant Frequencies	Travel Time (Gutenberg)	Travel Time (Trav.Spec.)	Center Time
ScS (V)	.070 - .560 c.p.s.	17.25 Min.	18.19 Min.	2.07 Min. (U-D)
ScS (H)	.000 - .560	-----	18.62	2.50 (E-W)
?	.000 - .560	-----	29.10	12.98

TABLE XIII

Santa Cruz Islands Quake

Phase	Dominant Frequencies	Travel Time (Gutenberg)	Travel Time (Trav.Spec.)	Center Time
PP)	c.p.s.	(20.90)		
PKS)	.063- .084	(21.65) Min.	21.54 Min.	5.71 Min.
PPP	.063- .070	23.00	22.91	7.08
PS	.035- .077	30.85	30.89	15.06
PPS	.028- .063	32.03	31.77	15.94
PPPS	.000- .028	33.05	33.53	17.70
PSPS	.021- .049	37.95	38.25	22.42
ScSScS	.000- .021	40.60	40.61	24.78
SSS	.021- .042	41.85	42.09	26.26

With regard to the Dominican Republic Quake we found it difficult to ascertain the nature of a prominent wide frequency band emergy pulse which occurred on both U-D and E-W component records at 20:15:03 G.S.T. (Center Time 11.46 Min.). At first we thought that might be some reflected S phase caused by the P but the frequency spectrum of this pulse contains energy outside that of the P for this quake. Comparison of its spectrum with that of the S showed striking similarity between the two and the likeness seemed to suggest a reflection of the S, however, the travel time charts indicate nothing of such a nature arriving at the time in question. It seems that all that one has left to explain the occurrence of this phase is the Love wave, when one takes into account the band of frequencies involved. Further consideration of the difference in time between the arrival time of this phase and the Love wave strongly suggests the generation of a "Rayleigh" wave by the Love wave as it strikes the continent. The nature of the components of the motion indicate retrograde elliptical motion in a plane, perpendicular to the direction of travel- a screw-like motion - and seemingly detracts from this notion. Definite statements, however, must necessarily await further mathematical analysis and observation.

(b) The P-Wave

A generally accepted definition of earthquake magnitudes proposed by Richter (see Bullen) is the following: magnitude is the logarithm (to the base ten) of the maximum amplitude (measured in microns) traced on a seismogram by a standard short-period seismograph (free period 0.8 sec.; statical magnification 2800; damping coef. o.8), distant 100 kms. from the epicenter. Empirical tables for quakes of normal depth have been set up to enable reduction of amplitudes measured at various distances to the expected amplitudes at the standard 100 kms. These tables have been built on the assumption that the ratio of the maximum amplitudes at two given distances is the same for all earthquakes. Since these assumptions are not strictly true many observers have advocated the use of P-wave period in establishing the magnitude of an earthquake.

For instance, Kanai, et al. (1953) claim that "the amplitude of earthquake motion is considerably influenced by direction, damping, construction of earth crust and property of the ground near the observation point. Then it is easily seen that the relationship between the amplitudes at earthquake origin and at the observation point is not so simple. Therefore, in general, there will be a considerable inaccuracy in determining the energy or magnitude of earthquake by using the amplitude of earthquake motion even if we adopt the initial motion of P-waves which is considered to communicate the characteristics of seismic waves generated at the earthquake origin comparatively well.

"On the contrary, the period of seismic waves, particularly that of the initial motion of P-waves, can be considered to keep the characteristics of the waves unchanged from the earthquake origin to the observation point. Consequently, if the relation between the period and the energy can be clarified, the energy or the magnitude of earthquake will be given more accurately by usihg the period of seismograms than by using the amplitude."

More recently Kanai, Osada and Yoshizawa (1953) have inaugurated and extended studies of the relation between the period and amplitude of the P-wave. The results of their studies indicate among other things that the amplitude is roughly proportional to the square of the period of the motion of this wave. It may be reasoned then, that for quakes of similar magnitude, there should be a noticeable decrease in period with increase in epicentral distance.

It would seem, then, that if any determinations were to be made from the period of the P-wave exact analysis of same for frequency content should be preferred over any rough determinations which might be made by measurement with dividers and time scale. As we have shown previously the frequency estimates of a function of time are obtained by determining the auto-correlation of this function and performing a cosine transformation on this to obtain the power density spectrum.

In an effort, therefore, to test and display the usefulness of spectrum analysis in this regard we have determined the power spectra of four P waves from quakes of various epicentral distance. The data pertaining to these quakes and listed below were obtained from the United States Coast & Geodetic Survey. The results of our calculations are depicted in figures (15) - (18).

I	Date	- May 31, 1953
	Epicenter	- Dominican Republic 20°N - 70½°W
	Time	- 19:58:35 G.S.T.
	Magnitude	- 7 (Pas.) 7½ (Berk)
	Distance	- 2,500 kms
II	Date	- Dec. 12, 1953
	Epicenter	- Peru 3½°S, 81° W
	Time	- 17:31:22 G.S.T.
	Magnitude	- 7-3/4 (Pas.)
	Distance	- 5,150 kms
III	Date	- Dec. 7, 1953
	Epicenter	- N. Chile 22°S, 68½° W
	Time	- 02:05:37 G.S.T.
	Magnitude	- 7¼ Pas. Depth - 100 kms
	Distance	- 7,220 Kms
IV	Date	- Sept. 23, 1953
	Epicenter	- N. Kurile Is. 50° N, 156° E
	Time	- 02:14:36 G.S.T.
	Magnitude	- 7 Pas 6¼ (Berk)
	Distance	- 8,650 kms

Comparison of the magnitudes of quakes I - III and the spectra of the corresponding P-wave shows that the expected decrease in period with distance for quakes of approximately the same magnitude is a fact for the predominent period involved, although quake IV seemingly indicates the opposite effect. The period of quake IV should have been the least. See Kanai (1953). It would seem that the only way to account for Quake III and IV having the same prevailing frequency would be to increase the magnitude of quake IV given by the Coast & Geodetic Survey. It may be, however, that effects other than energy play a more dominating role in this instance. For example Kanai, et al. have established the relation

$$a = \frac{T}{2.6}\sqrt{\frac{\mu}{\rho}}$$

where a is the radius of the origin (assumed spherical), T is the P-wave period measured at a station, and $\sqrt{\frac{\mu}{\rho}}$ is the shear wave velocity in the vicinity of the focus. It can be easily seen, then, that if the radius is assumed to be constant in all quakes, deeper foci will ~~have~~ GIVE rise to shorter P-wave periods.

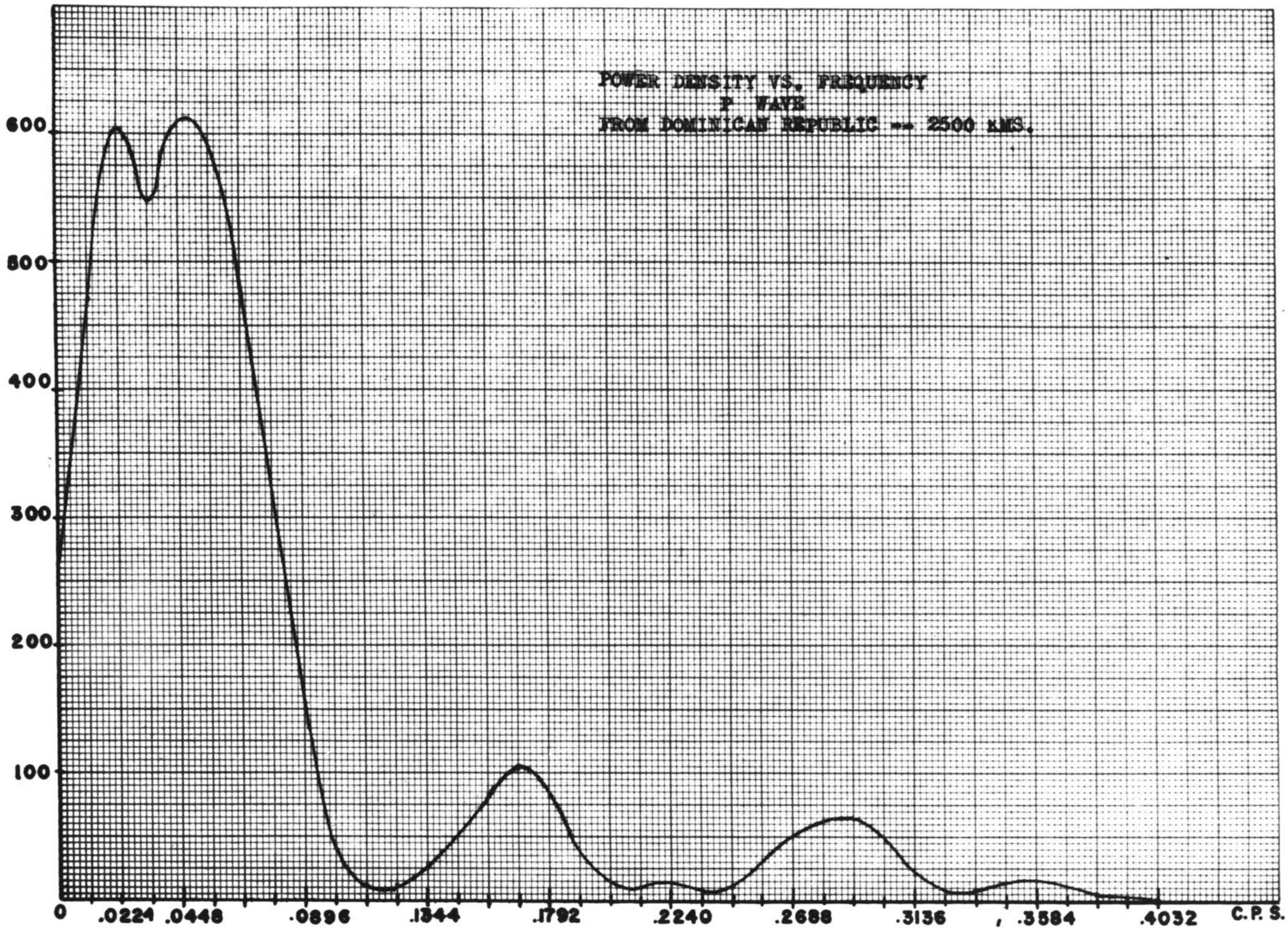

POWER DENSITY VS. FREQUENCY
P WAVE
FROM DOMINICAN REPUBLIC -- 2500 KMS.
600
500
400
300
200
100
0
.0224
.0448
.0896
.1344
.1792
.2240
.2688
.3136
.3584
.4032
C.P.S.

Fig. (15)

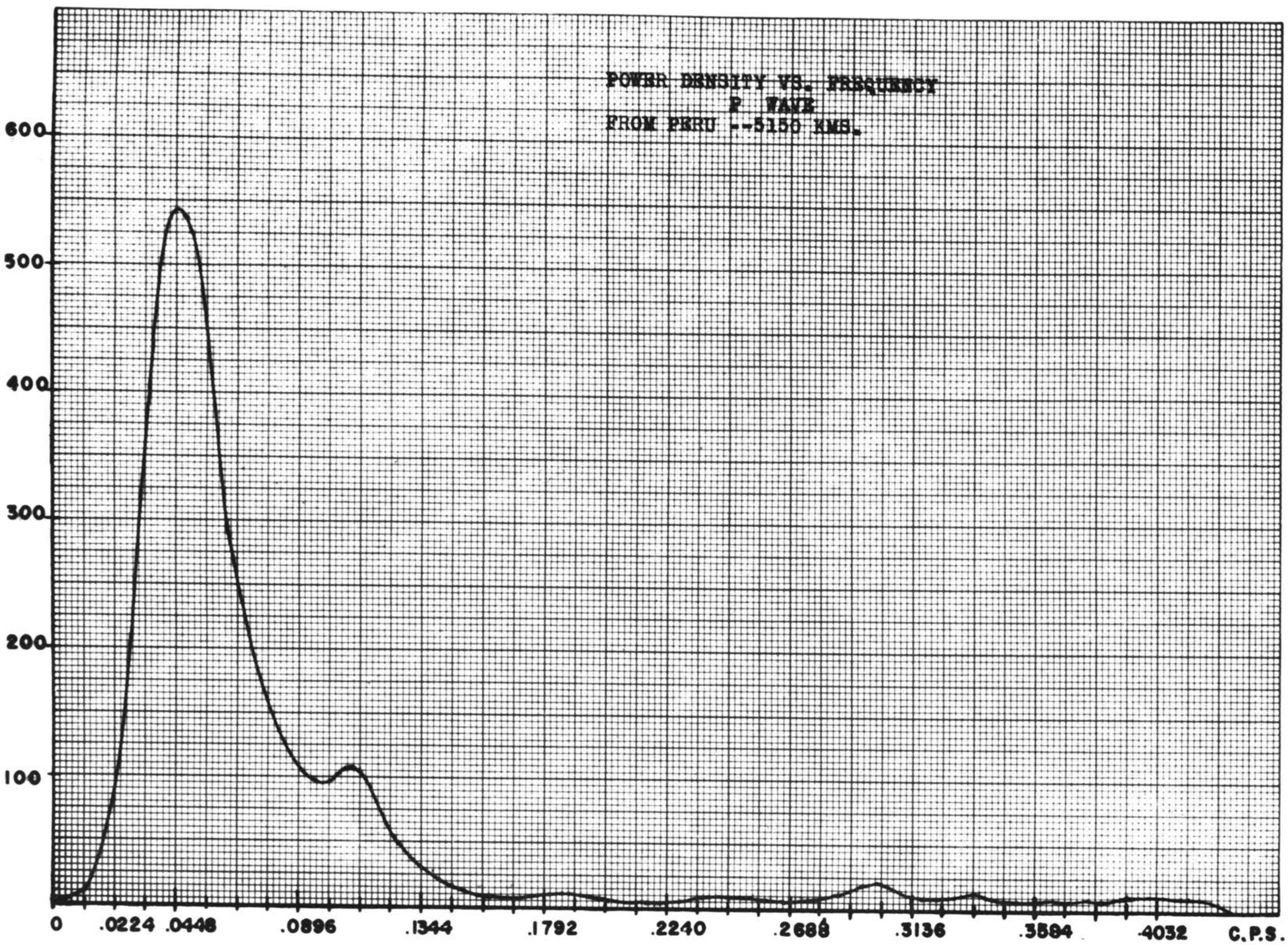
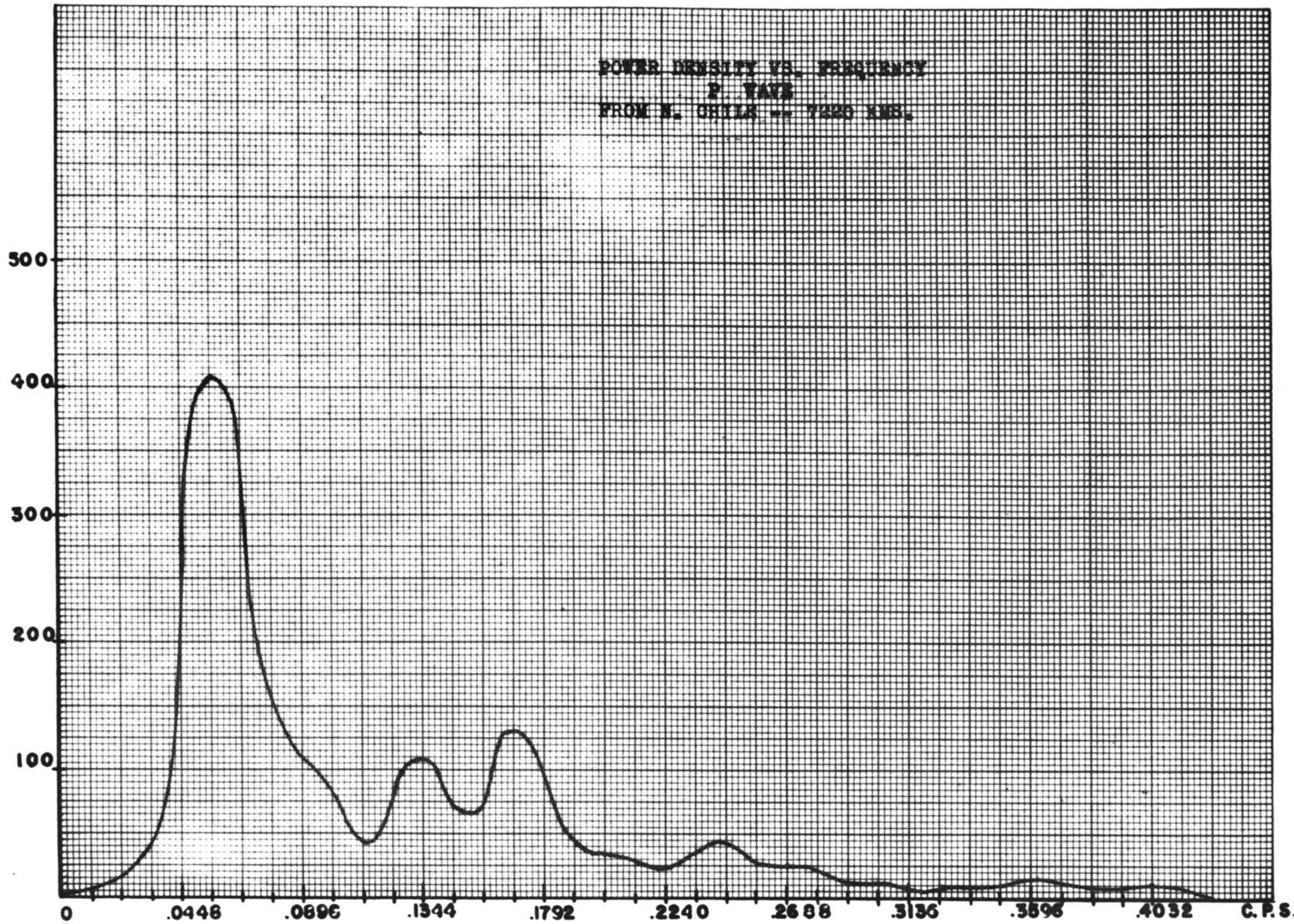

POWER DENSITY VS. FREQUENCY
P WAVE
FROM PERU --5150 KMS.
600
500
400
300
200
100
0
.0224
.0448
.0896
.1344
.1792
.2240
.2688
.3136
.3584
.4032
C.P.S.

Fig. (16)



POWER DENSITY VS. FREQUENCY
P. WAVE
FROM N. CHILE -- 7220 KMS.
500
400
300
200
100
0
.0448
.0896
.1344
.1792
.2240
.2688
.3136
.3596
.4032
C.P.S.

Fig. (17)

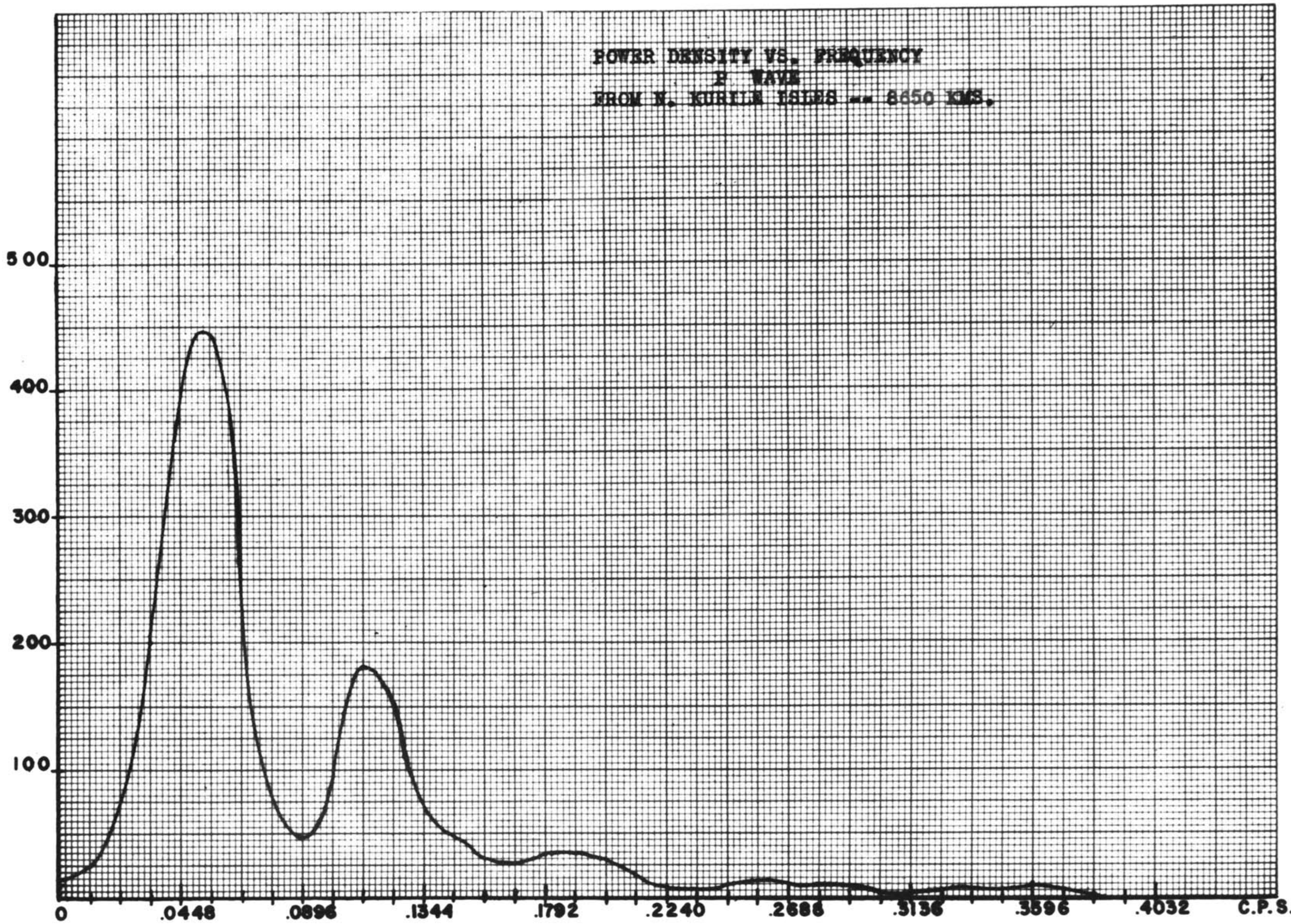

POWER DENSITY VS. FREQUENCY
P WAVE
FROM N. KURILE ISLES -- 8650 KMS.
500
400
300
200
100
0
.0448
.0896
.1344
.1792
.2240
.2688
.3136
.3596
.4032
C.P.S.

Fig. (18)

D.. The Possibility of Surface Wave Generation at the Edge of a Continent

In a previous section dealing with Rayleigh and Love wave dispersion from a quake occuring in the Santa Cruz Isles, we had occasion to mention the procedure we used in determination of groups which we believed to have been generated at the source. We repeat, that the continental dispersion for both types of surface waves determined from the Vancouver Is. quake was subtracted from all groups which evidenced themselves on the traveling spectra of the U-D and E-W components of the Santa Cruz Isles quake. Our method was to determine the continental travel time from our observed Love wave and Rayleigh wave (curve B) curves for a 4,650 km. path and to subtract these times from the center-times of the respective groups or pulses which we observed for the Santa Cruz quake, plate (10). This correction should then have given the "center-time" arrival time of the Rayleigh and Love groups at the Pacific edge of the continent. Our results for this computation are listed in Table XIII.

Such a listing of groups enabled us to determine fairly accurately which groups were from the source, for in most instances the Love center-time at this point would differ from that of the corresponding Rayleigh group by approximately three to four minutes, which roughly should be the case if the two traveled across the Pacific to this point.

It was noticed in the tabulation of these results that several Love and Rayleigh groups exhibited approximately the same center-time arrival time at this point. If we should arbitrarily coose our maximum error as being the time from the center-time of the interval to the extremities of the interval in question, namely, $\pm .590$ min. we then see that fairly good agreement is had in these instances. It may be possible to discredit this by better observations of surface wave dispersion across the continent, by considerations of chance, by identification of the groups in question as being something other than surface waves, or by determining the direction of approach of these waves as being inconsistent with a direction which is direct from the source to Weston. However, we do claim that the evidence clearly suggests the possibility of generation of Love waves by Rayleigh waves (or visa versa) as they strike the continent.

A firm basis for such a claim, of course, can only be established by further observations and theoretical considerations.

TABLE XIII A

Continental Travel Times (4,650 kms.)

GROUP (FREQUENCY)	LOVE WAVE	RAYLEIGH WAVE
.014 c.p.s.	20.1 min.	----- min.
.021	20.6	-----
.028	21.3	22.0
.035	21.8	22.9
.042	22.0	23.4
.049	22.3	23.7
.056	22.5	24.1
.063	22.6	24.2
.070	22.8	-----

TABLE XIII B

Time of Surface Waves at the Pacific Edge of the Continent

GROUP (FREQUENCY)	LOVE WAVE CENTER-TIME	A	RAYLEIGH WAVE CENTER-TIME	B
.028 c.p.s.	38.94 min.	17.64 min.	43.95 min.	21.95 min.
	42.88	21.18		
.035	45.73	23.93	45.13	22.23
	43.36	21.56		
	40.12?	18.32		
.042	46.32	24.32		
	42.48?	20.48	47.49	24.01
.049	49.56	27.02	49.56	25.86
	44.84?	22.30	50.74	27.04
.056	52.51	30.01	53.69	29.57
	48.38	25.88	55.46	31.36
	42.77	20.27		
.063	59.00	36.40	63.72	39.52
	48.09	25.49		
	42.77	20.17		
.056-.063	64.31	41.76	66.08	41.88
	65.49	42.74	69.03	44.83
	69.03	46.48	71.78	47.78
	74.34	51.79	76.11	51.93
.070	42.48	19.68	74.34	50.28
	47.20	24.40	81.42	57.12
	73.75	50.25		

A-- Love center-time minus continental travel time.
B-- Rayleigh " " " " " .

E... A Study of Microseisms

Recently, Dr. K. E. Haq (1954) in a doctor's thesis at M.I.T. has clearly shown that the microseisms are generated by standing ocean waves which are formed by a system of traveling waves of approximately the same period coming from opposite directions, as originally proposed by Longuet-Higgins (1950). His investigations and arguments, in addition, completely negate microseisms being generated by the pounding of surf against a rocky coast, which method has been vigorously defended by Gutenberg and others.

Miche (1944) has shown from a theoretical study of wave motion that the ocean bottom beneath a train of standing waves of the form

$$\xi = a \cos kx \cos \beta t + O(a^2)$$

where

$$\beta^2 = gk \tan kh$$

$$\lambda = \frac{2\pi}{k}$$

$$Z = \xi \quad \ldots\ldots \quad \text{the ocean surface}$$

$$Z = -h \quad \ldots\ldots \quad \text{the ocean bottom}$$

will be subjected to a fluctuating mean pressure,

$$\frac{P - P_o}{\rho} = gh - \frac{1}{2} a^2 \beta^2 \cos 2\beta t + O(a^3)$$

whose frequency is twice the frequency of the waves on the surface and whose amplitude is proportional to the square of the amplitude of the stationary waves on the surface. Haq (1954) has already shown this frequency relationship to be a matter of fact in his investigations of the power density spectra calculated from microseisms recorded at Weston Observatory, Weston, Mass. and from swell records obtained from Woods Hole, Mass. and Gilgo, L. I.; and also by comparing periods of the microseisms with those of the ocean waves reported from the weather ship 4YH, located 36°N 70°W.

Theoretical considerations show that if the depth of water

$$h = \lfloor \frac{n}{2} + \frac{1}{4} \rfloor$$

where h ....depth of water

λ ....wave length of compression al waves in water

n ....0, 1, 2, ...

the amplitude of microseisms may increase by a factor of 4 - 5 due to resonance effects. We can see, then, that if standing waves are situated over an ocean floor which is variable in depth, many frequencies will give rise to resonance and the generated microseisms will contain wide band of frequencies. Should the standing waves occur in those regions of the ocean whose depth is constant the microseisms arising therefrom will exhibit narrow band frequency spectra. Further speculation seems to indicate that if the area of origin is near to the recording station those frequencies which are not those of resonance and which are relatively of minor importance will also contribute to the broadness of the microseisms frequency spectra. As this generating area becomes further removed from the station those frequencies, and especially the higher ones, which are not favorable to resonance, will become of increasingly less importance due to effects of decay and absorption.

In view of the aforementioned considerations we would in actuality expect to find for a storm proceeding from the land but over the continental shelf into deeper and deeper water, microseisms frequency spectra displaying in time increasing narrowness, and increasing predominance of the lower frequencies. It has been Haq's observation "that at the beginning of a storm, microseisms are usually low in amplitude and very irregular in character, at which times the fronts are usually on the shelf or at the edge of it. As the fronts move into the deep water regions, where the depth is more uniform, the microseisms increase in amplitude and look like regular pulses. This shows that the depth of water in the generating area has a significant effect on the nature of microseisms." (Above is evident on plate(11).)

We have in an effort to confirm the veracity of these speculations, calculated the traveling amplitude spectra -- of the two minute intervals -- and power density spectra -- of thirty second intervals -- of four samples (separated by approximately twelve hours) of microseisms. The storm in question was recorded at Weston December 11th through December 13th, 1953, and the seismograms from which the samples were taken were the long period verticals. Photostatic reproductions of those portions of the microseism record utilized may be found on plate (11); the actual two minute samples employed in this study occur between the superimposed brackets. The progress of the storm's intensity as far as microseisms and wave heights at 4 Y H are concerned are depicted in fig. (19) (Courtesy of K.E. Haq).

In general, the storm was caused by the eastward passage of a cold front from the coast of the United States into the Atlantic Ocean; the actual details of meteorology, and the method of generation of standing ocean waves in this storm have been adequately described by Haq in his "Case 1". According to Haq, standing waves were generated on the shelf at approximately 16:00 G.S.T. Dec. 11th, over the slope at about 4:00 G.S.T. Dec. 12th, in deeper regions at 16:00 G.S.T. Dec. 12th, and farther into the ocean at approximately 4:00 G.S.T. Dec. 13th.

The traveling spectra figs. (20) through (21) computed in the vicinity of these respective times show that, as time proceeds, the spectra become more sharply peaked. Figs. (22) -- (25) depict this fact in two dimensional form. The two resonant peaks at .225 and .36 C.P.S. at 16:00 Dec. 11th would seem to indicate that the standing waves had proceeded farther out onto the shelf than Haq had expected from meteorological considerations at that time. At 4:00 Dec. 12th we see that the major contribution of frequencies exists in the band .135 -- .36 C.P.S. with a maximum at .247 C.P.S. At 16:00 G.S.T. Dec. 12th the band of frequencies is .147 -- .36 C.P.S. with the major contribution from .225 C.P.S. In fig. (25) we see that the same band of frequencies,-- with the same maximum -- occurs at 4:00 G.S.T. Dec. 13th, although the contribution from higher frequencies (.63 - .81 C.P.S.) is decidedly less.

This study shows rather conclusively that in one instance, the resonating phenomena expected from considerations of the standing wave generation of microseism is indeed a fact. In addition to this, if the nature of microseisms and their spectra are studied as the storm proceeds we see that additional evidence is had for the standing wave theory of generation. For at the beginning of the storm the microseisms are irregular and the spectra are of the wide band type as the storm proceeds into deep water the microseisms appear as pulses and the derived spectra are more narrow with major contributions at lower frequencies. Such phenomena could hardly be expected from surf action on a rocky coast. As a matter of fact no change would be evidenced, as already shown by Haq's spectra on this matter.

MICROSEISMS DEC.11 – 13, 1953 AT WESTON

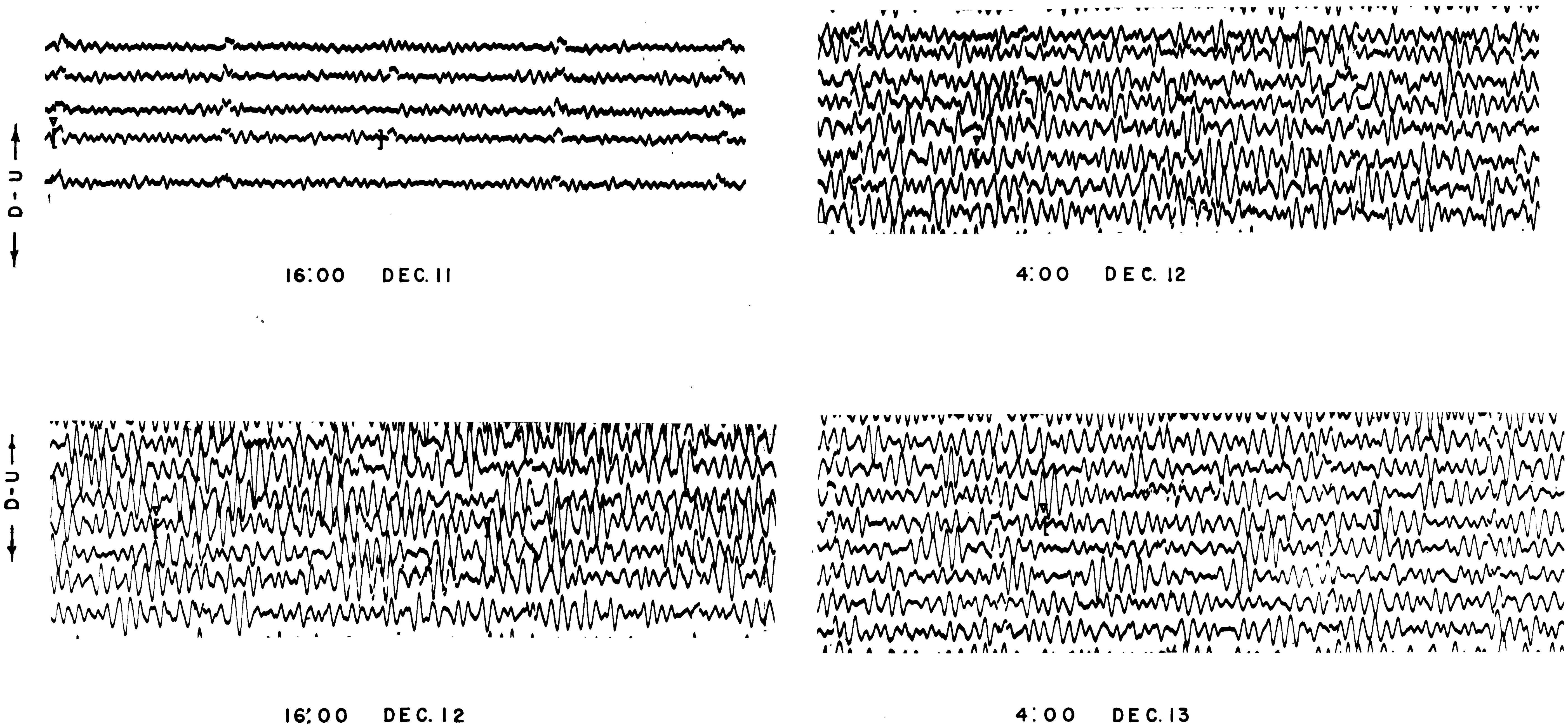
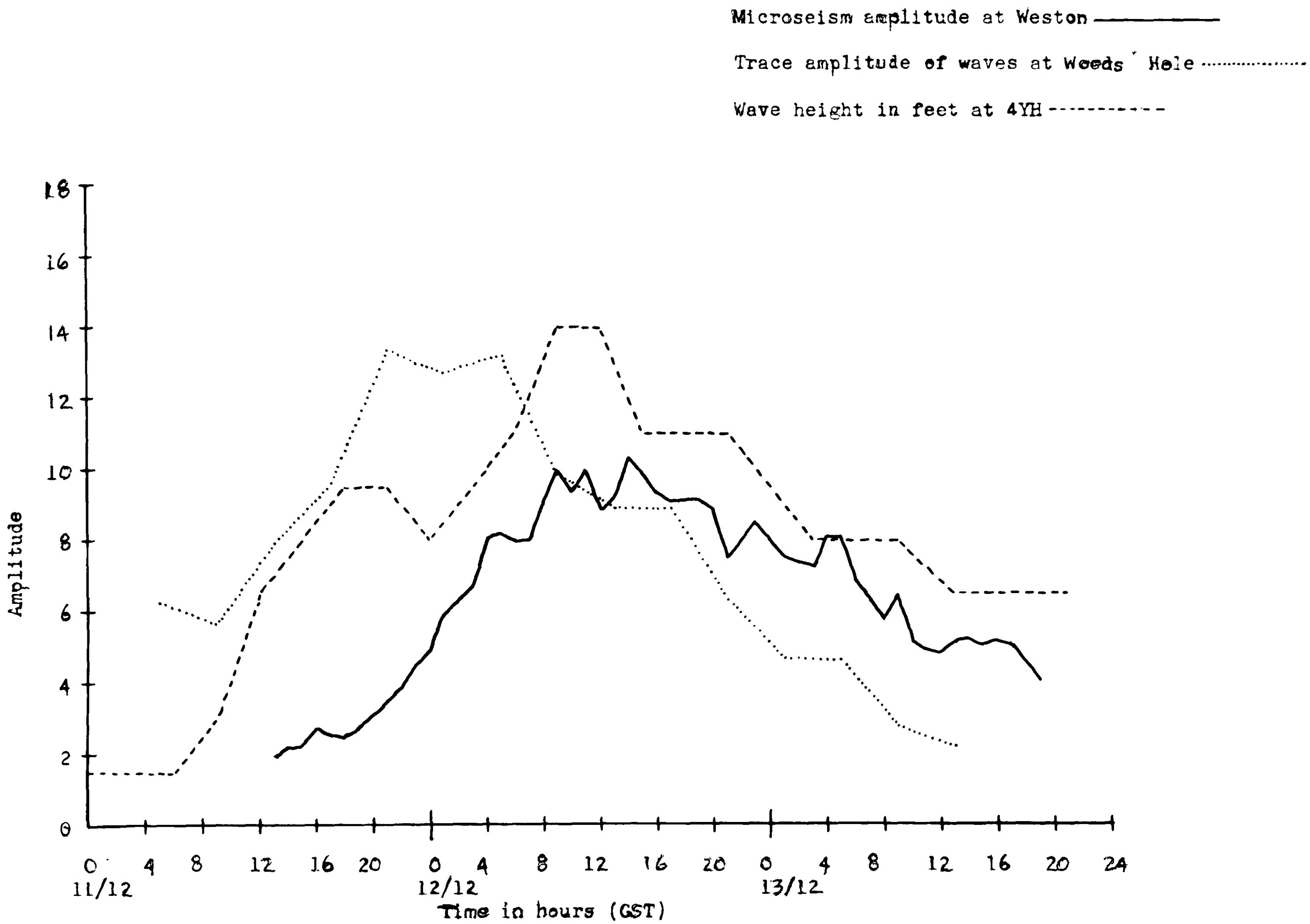
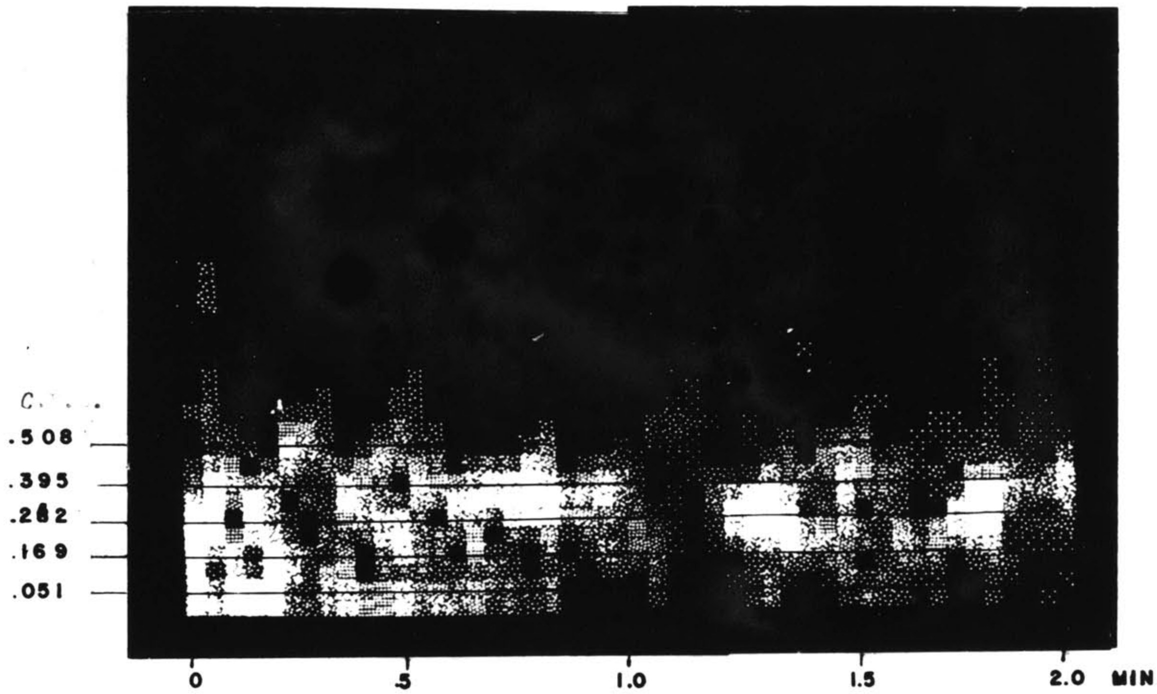


PLATE 11

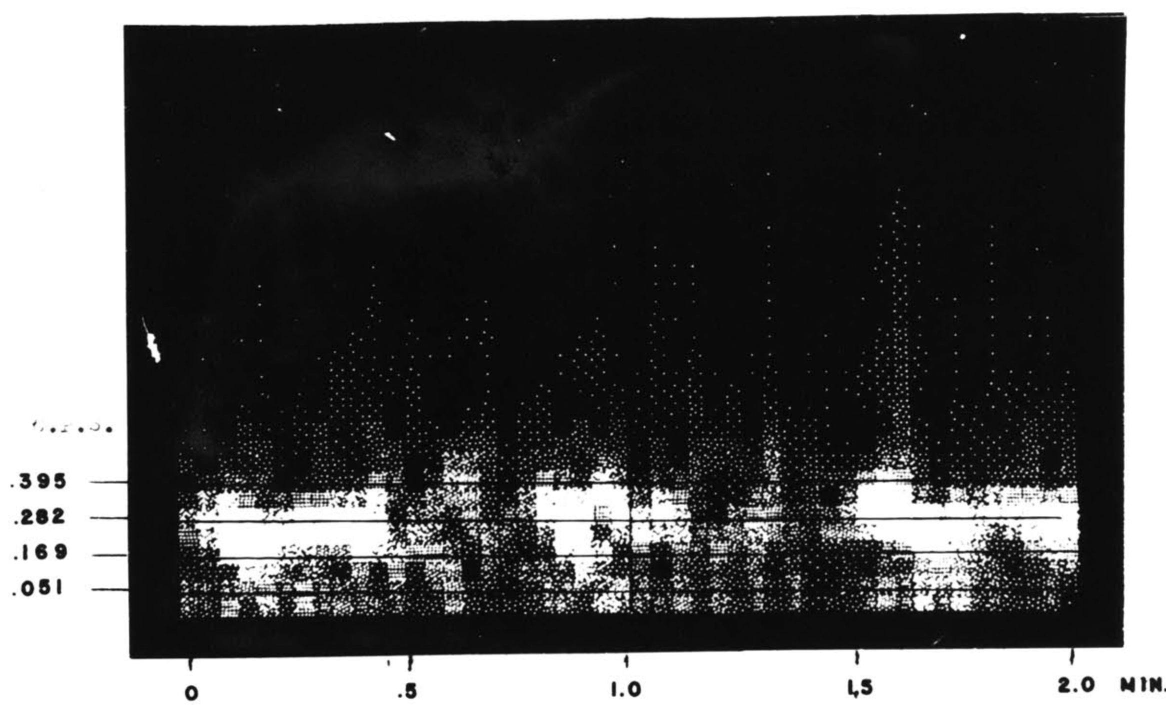
Fig. (19)



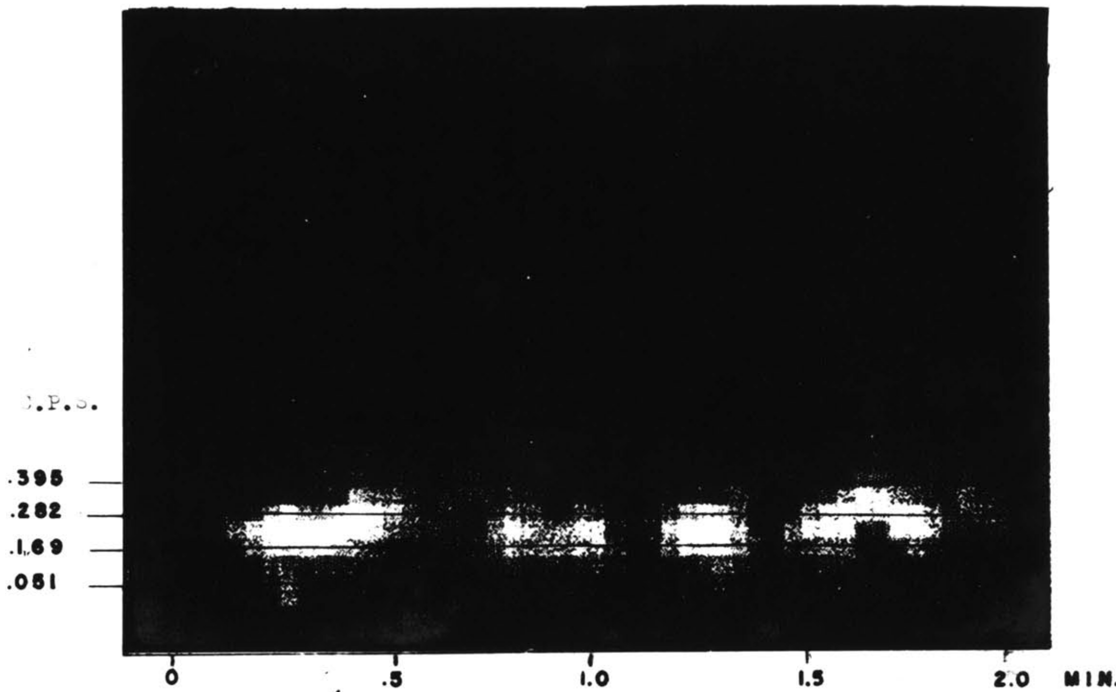
Graph 1.



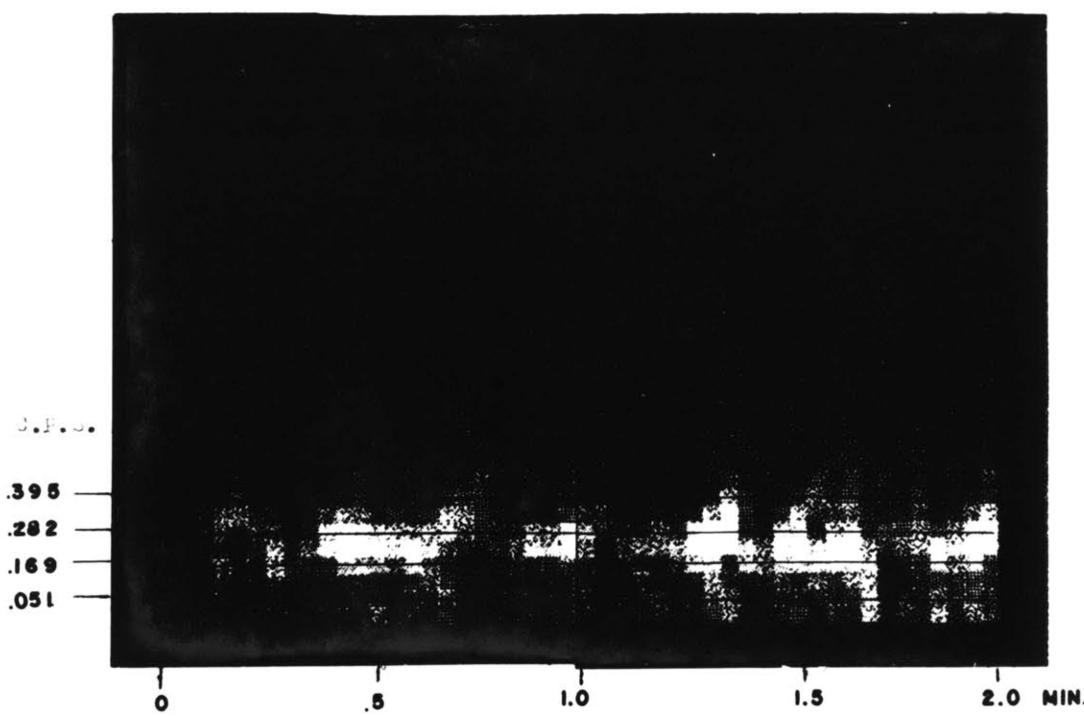
Traveling Spectra of a Two Minute Interval in the Vicinity of 16:00 (GST) Dec. 11, 1953.



Traveling Spectra of a Two Minute Interval in the Vicinity of 4:00 (GST) Dec. 12, 1953.



Traveling Spectra of a Two Minute Interval in the
Vicinity of 16:00 (GST) Dec. 12, 1953.



Traveling Spectra of a Two Minute Interval in the
Vicinity of 4:00 Dec. 13, 1953.

Fig. (21)

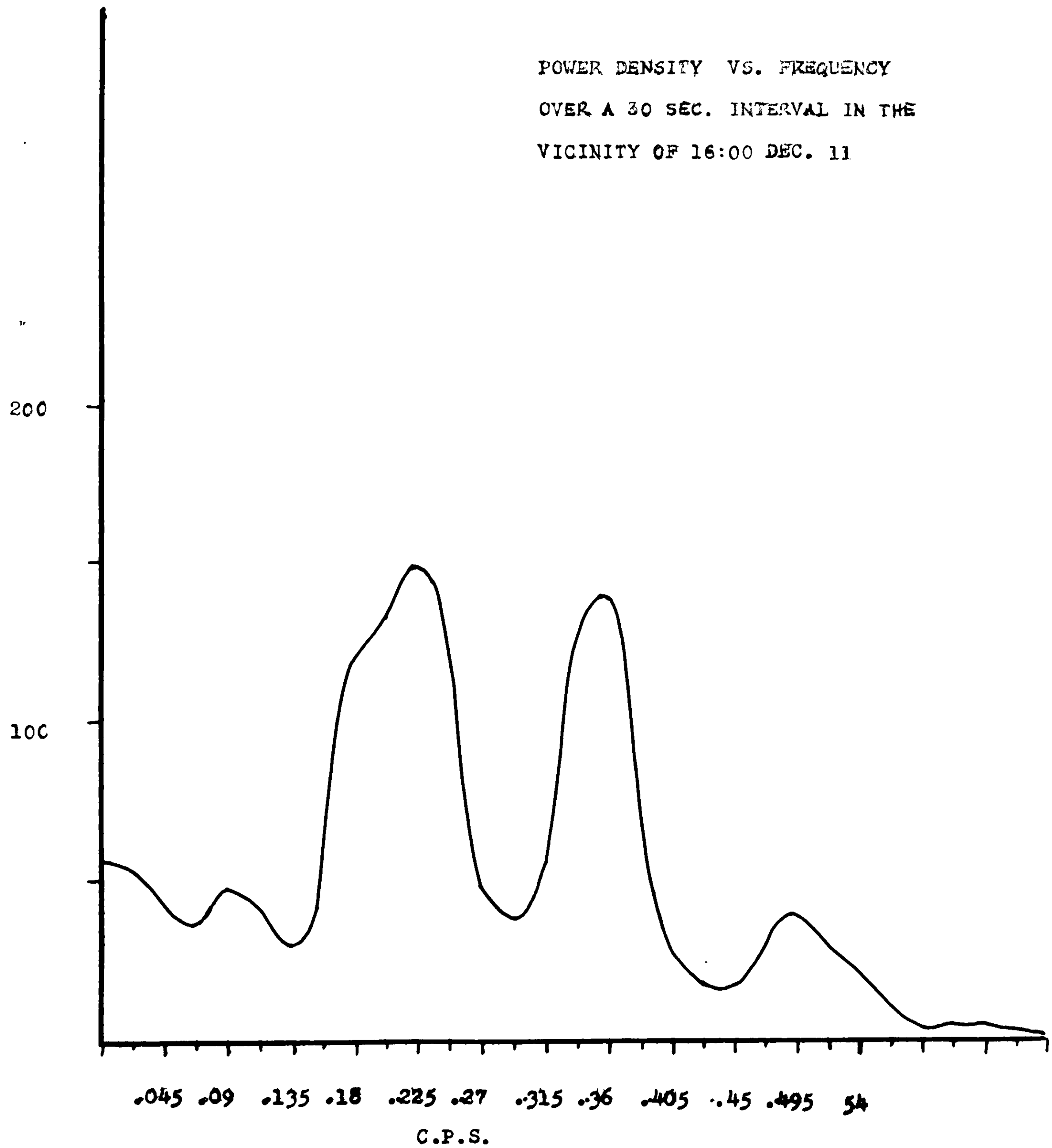

POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 16:00 DEC. 11
200
100
.045 .09 .135 .18 .225 .27 .315 .36 .405 .45 .495 54
C.P.S.

Fig. (22)

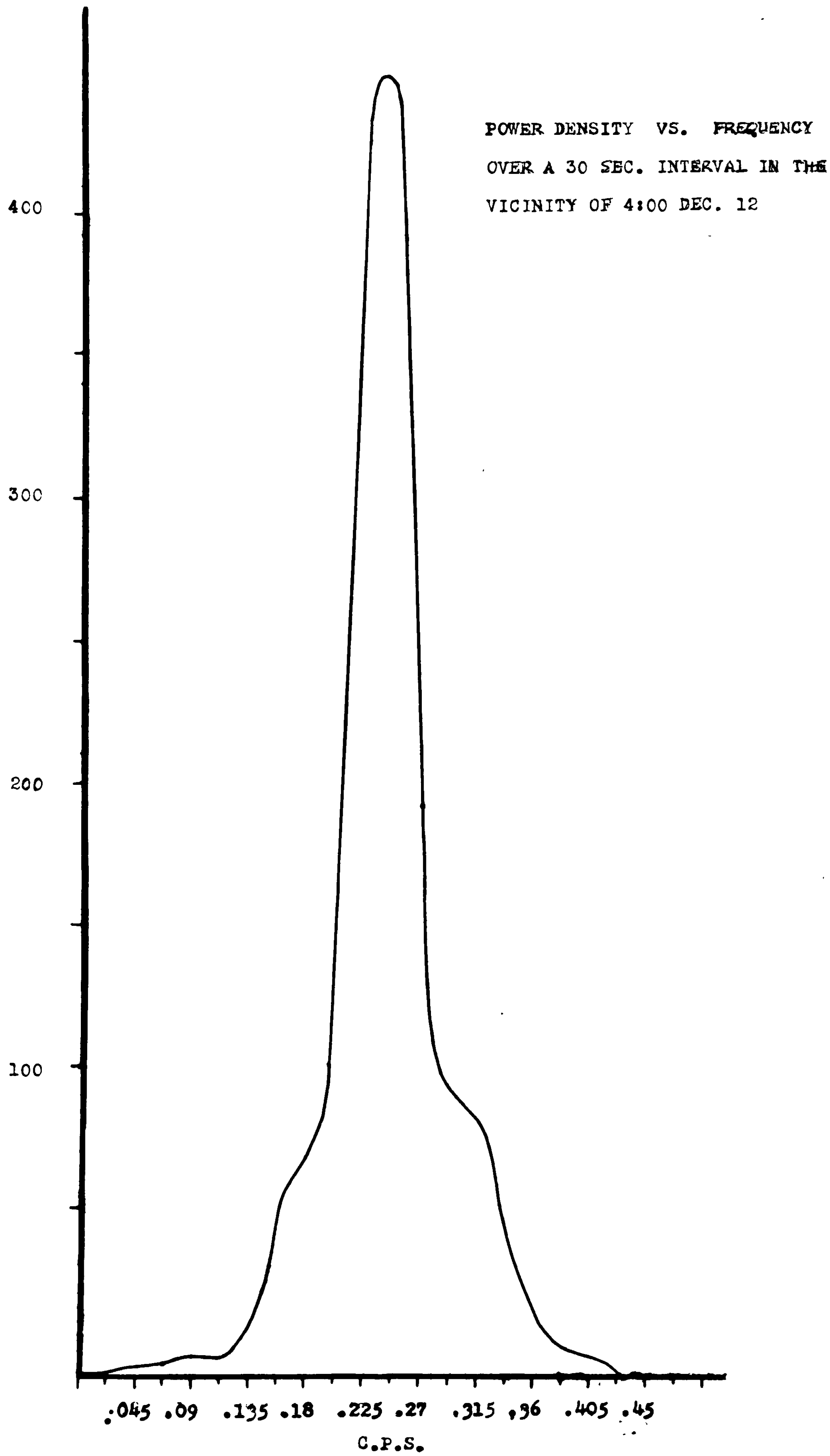

POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 4:00 DEC. 12
400
300
200
100
.045 .09 .135 .18 .225 .27 .315 .36 .405 .45
C.P.S.

Fig. (23)

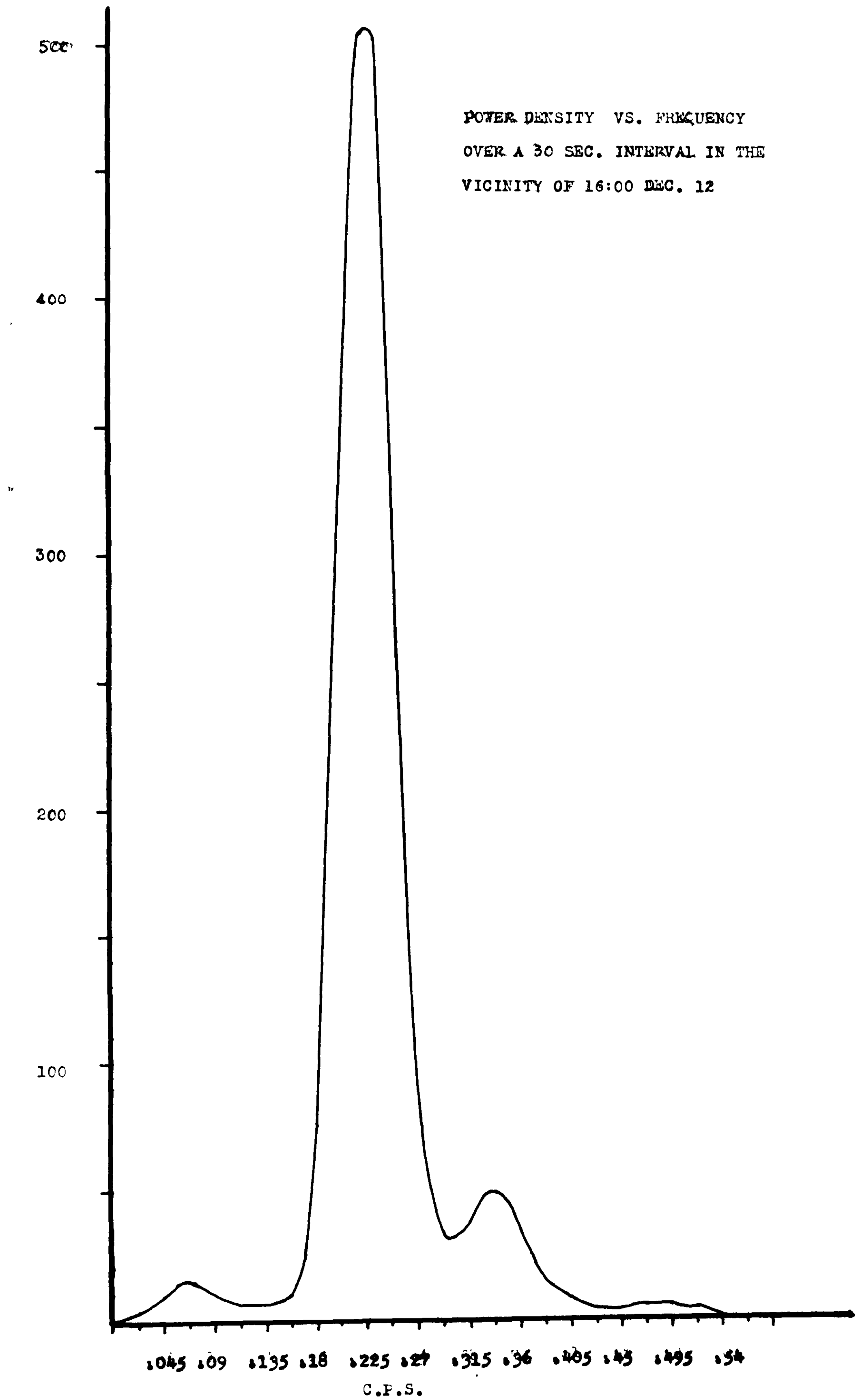

POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 16:00 DEC. 12
500
400
300
200
100
.045
.09
.135
.18
.225
.27
.315
.36
.405
.45
.495
.54
C.P.S.

Fig. (24)

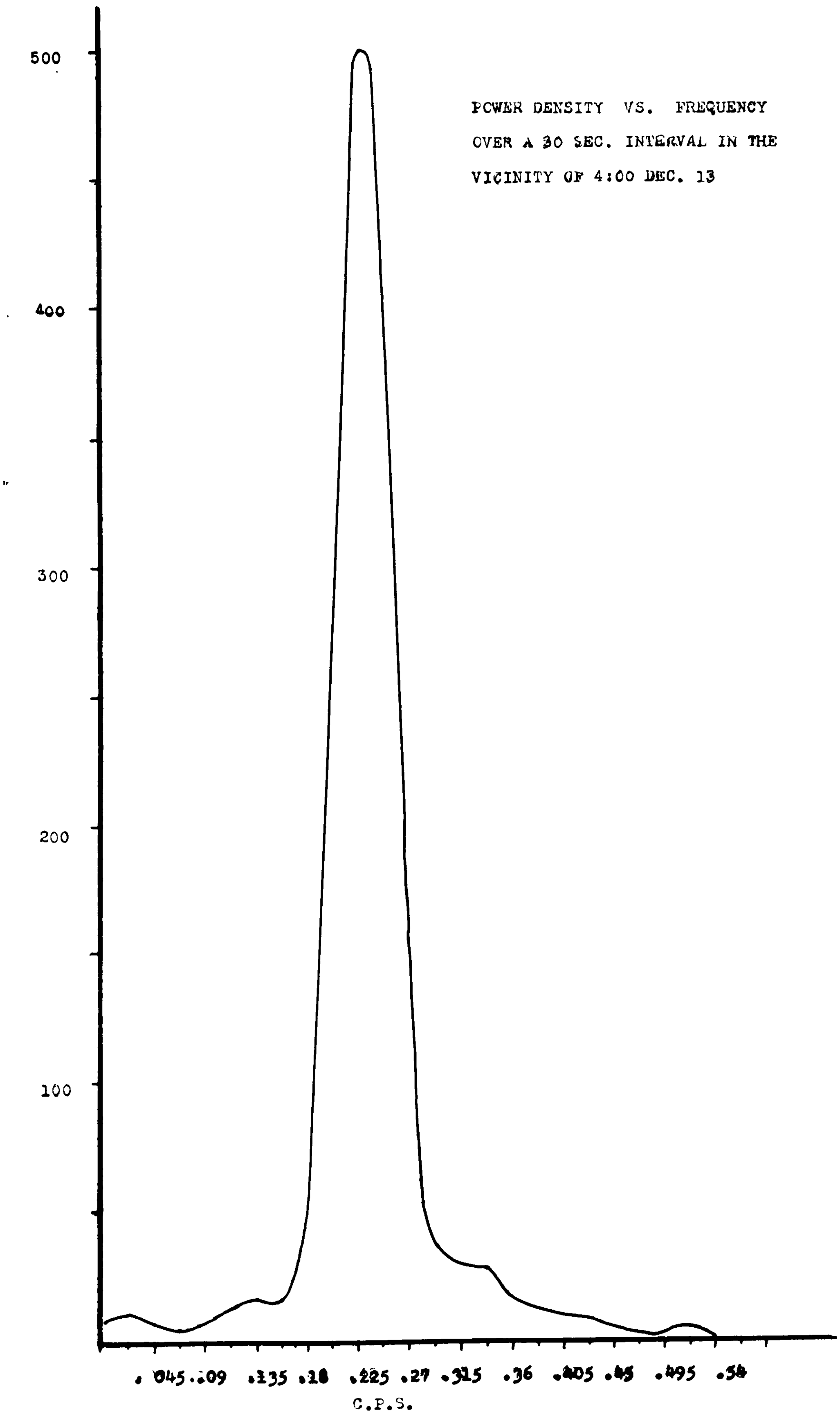

500
400
300
200
100
POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 4:00 DEC. 13
.045 .09 .135 .18 .225 .27 .315 .36 .405 .45 .495 .54
C.P.S.

Fig. (25)

F... <u>The Phase Spectra of Seismograms</u>.

We would, at this time, like to report that the high hopes we entertained for traveling phase spectra, which we have already described in the first chapter, did not in actuality materialize. Possibly improvements of our techniques in this respect or applying it to other branches of geophysics may bear more fruit. For example, studies of the phase relations between variations in telluric current and the earth's magnetic field as suggested by Cagniard (1953) and Wait (1954) may be profitable.

We present, nevertheless, the results obtained for traveling phase spectra in one instance from our analysis of a quake which occurred in the N.Gulf of California (already described) in Plate A solely for the purposes of illustration.

TRAVELING SPECTRA (PHASE) - N. GULF CALIFORNIA QUAKE

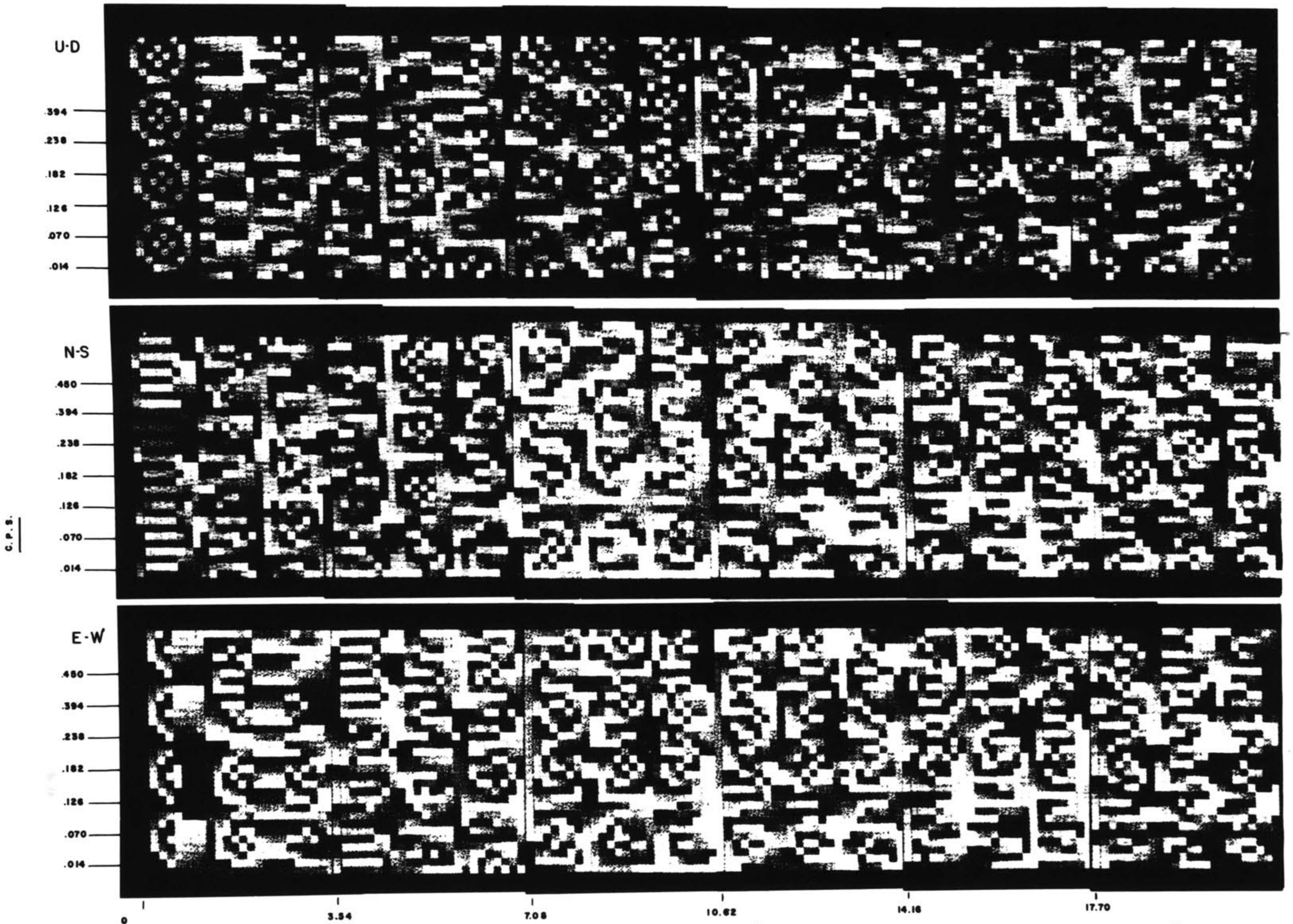


PLATE A

G... A Study of Seismic Prospecting Records

The success achieved in a foregoing section of correlating high amplitude spectra pulses with reflected and refracted energy of P and S phases of earthquakes suggests the possibility of using such means as travelling spectra to ascertain the presence of reflections not visible on prospecting seismograms. It seems doubtful at the present whether such a procedure would be able to pinpoint a reflection to the millisecond, but it would be of use in some cases in determining positions of the record which do not constitute reflected energy. To these regions then it would be possible to fit a linear operator* for purposes of filtering the record to better enhance the reflections. Thus far the determination of the regions of the record over which such operators are fitted has been rather arbitrary.

All the records which we have analyzed in this section have had data points read every 2.5 milliseconds. The travelling spectra have been calculated from successive forty point intervals overlapping by 75%. It is possible that such a technique could be enlarged on to bring out better portions of the record consisting of reflected energy, times of arrival and characteristics of each reflection, by taking shorter intervals with more closely read points, by overlapping intervals a greater amount, by investigating high frequency contribution, etc.

(*) The Geophysical Analysis Group of M.I.T. has devoted much attention and research in this direction in recent years. Their reports #1--#5 and the paper by Wadsworth, et al.(1953) explain this research quite thoroughly.

However, we will present our results that have been obtained to date for three such records.

(1) Presentation of the Results.

(a) A Record Containing Many Visible Reflections

Recently the Atlantic Refining Co. has supplied a number of records from Roosevelt County, New Mexico to the Geophysical Analysis Group for analysis. All the records exhibit a large number of reflections. The record shosen here was M.I.T. Record 7.4.

Twenty equally spaced groups of three seismometers each were used in a straight line in each spread, the interval between groups being equal to the group length. The frequency response curve of the amplifier and filter used on these records is given in fig. (26). Mixing in this record consisted of the addition of one half the output of each group of three seismometers connected in parallel to the output of the group next farthest away from the shot point. Each trace represents the recording of the output of such a combination. A reproduction of Record 7.4 may be found in Plate (12).

The results of a velocity survey taken in the locality are given in fig. (27). There is also given in this figure the tops of geologic formations present in this area.

The results of our computation for traces 1, 4, 7, 10, 13, 16 and 19 depicted photographically on plate (13).

We have also actually contoured the frequency time plot of trace 1 for purposes of comparison with the corresponding density plot. This contour is found on fig. (28).

In the following table we have listed the times of arrival of the reflections obtained from the record, together with traces whose traveling spectra seem to put the reflection into evidence by large power contribution at a particular time.

TABLE XIV

TRACES WHOSE TRAVELING SPECTRA EXHIBIT REFLECTIONS BEST

Reflection Time	Trace
.690 sec.	T - 1, 7, 10, 16
.820 -.830	T - 1, 4, 7, 10, 13, 19
.880	T - 1, 13, 19
.970	T - 1, 2, 7, 10, 13, 16, 19
1.030	T - 13, 16, 19
1.110 - 1.120	T - 1, 4, 13, 16, 19
1.180	T - 1, 4, 7, 10, 13
1.260	T - 7, 10, 13, 16, 19
1.330 - 1.350	T - 19
1.360	T - 4, 7
1.380	T - 1, 4, 7, 10, 13, 16, 19
1.430	T - 13, 16, 19
1.800	T - 10, 13, 19

(b) Record Containing Few Visible Reflections

This record was supplied by Magnolia Petroleum Co. to the Geophysical Analysis Group for analysis, the results of which have already been reported by Wadsworth, Robinson, et al. (1953). The only information pertaining to this record which is designated as M.I.T. Record No. 1 or 10.1 may be found on the reproduced record, Plate (14).

The traveling spectra for traces 1 - 6 of this record may be found on plate (15). Our ability to determine reflected energy by means of these analysis is recorded in the following table, which, as in the foregoing section, displays those traces from which spectral analysis was best in this regard. Reflection times were picked by Magnolia and are shown on plate (14).

TABLE XV

TRACES WHOSE TRAVELING SPECTRA EXHIBIT REFLECTIONS BEST

Reflection Time	Traces
.51 - .54 sec.	T - 1 to 6
1.00 -1.04	T - 1 to 6
1.16 -1.24	T - 1 to 6

(c) Record Containing No Visible Reflection

This record was also furnished to the Geophysical Analysis Group by Magnolia Petroleum Co. from a prospect in HendersonCo., Texas. The charge used was 5 lbs. at 295 ft. Each trace represents the recording of nine geophones. The record is shown in plate (16) and is designated as 10.9

Reflection times on the top trace T 1 are marked by •. The times were determined by Magnolia from a different shooting procedure.

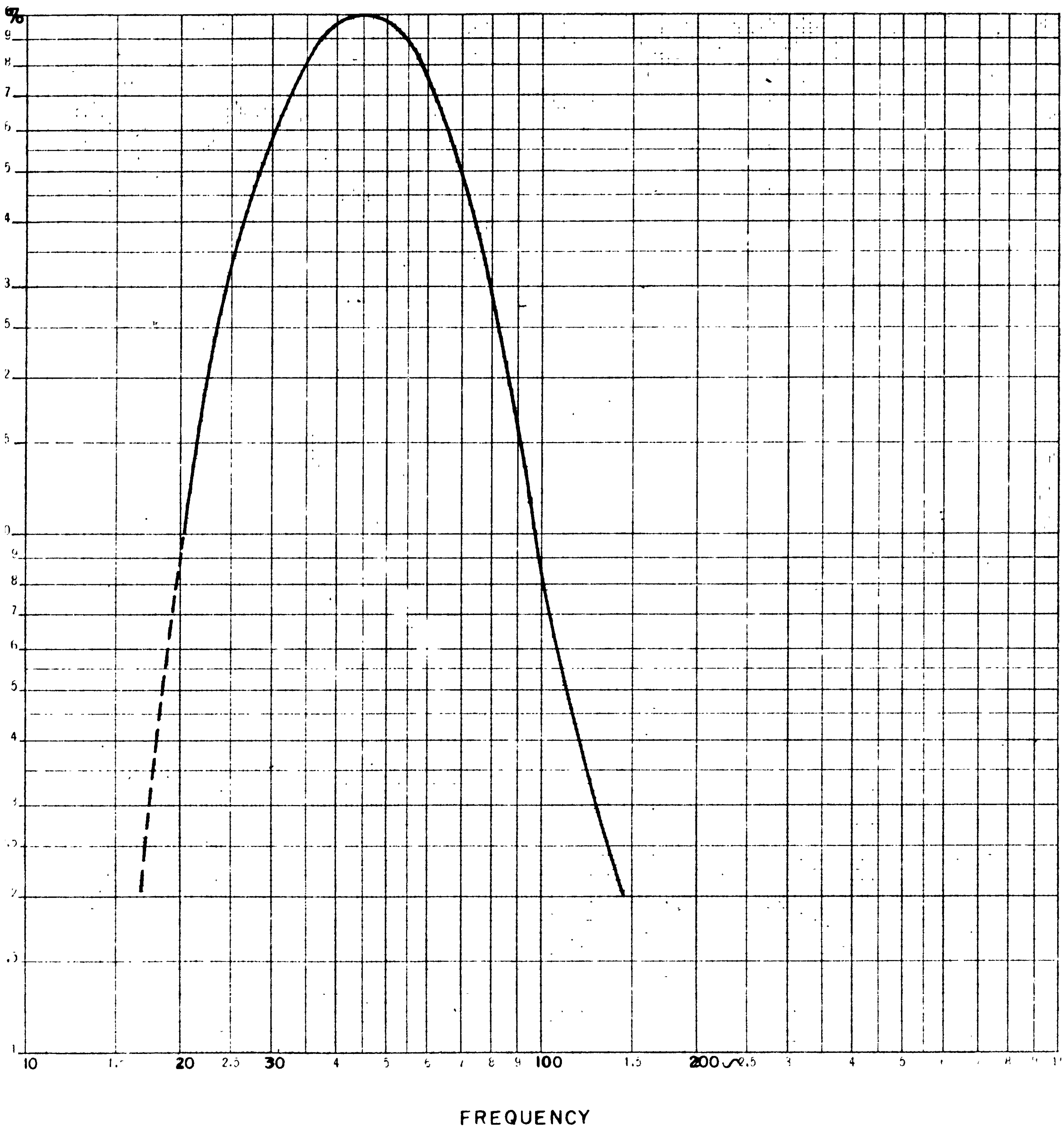
Our traveling spectra for the top four traces are shown in plate (17). A comparison of the traveling spectra and the reflection times on plate (16) shows that all five are displayed fairly well by the spectral analysis of traces

1 and 3 and extremely well on traces 2 and 4.

It is evident from our foregoing analysis that much can still be done to perfect our method, if it is, in the future to be seriously considered as a means to assist in the determination of reflected or refracted energy. What has transpired thus far should be regarded as experimental, the hope being that what has been presented here will stimulate more research in this direction.

FREQUENCY RESPONSE
25A AMPLIFIER
FILTER SETTING: 35-55

RECORDS 7.1 TO 7.6



[redacted]

Fig. (26)

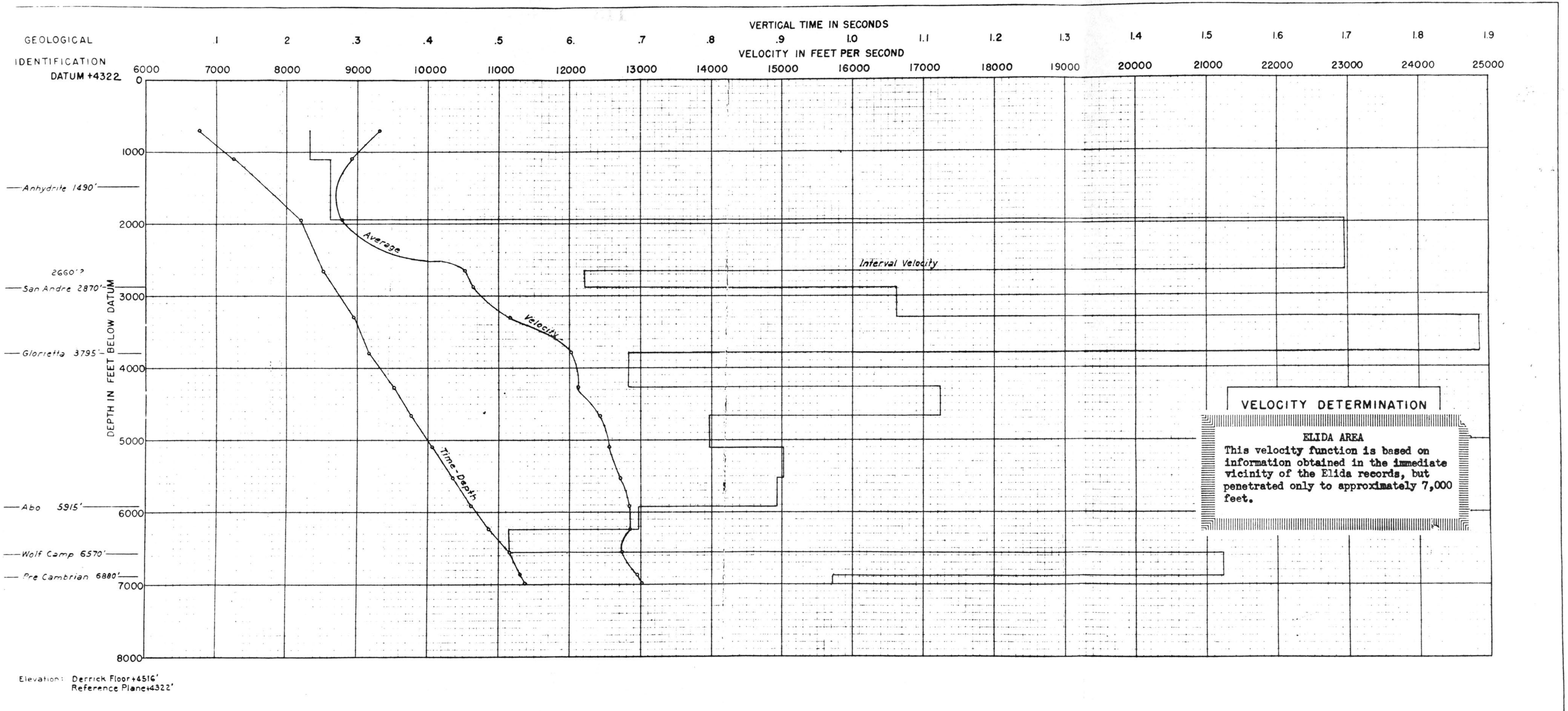


FIGURE V

Fig. (27)

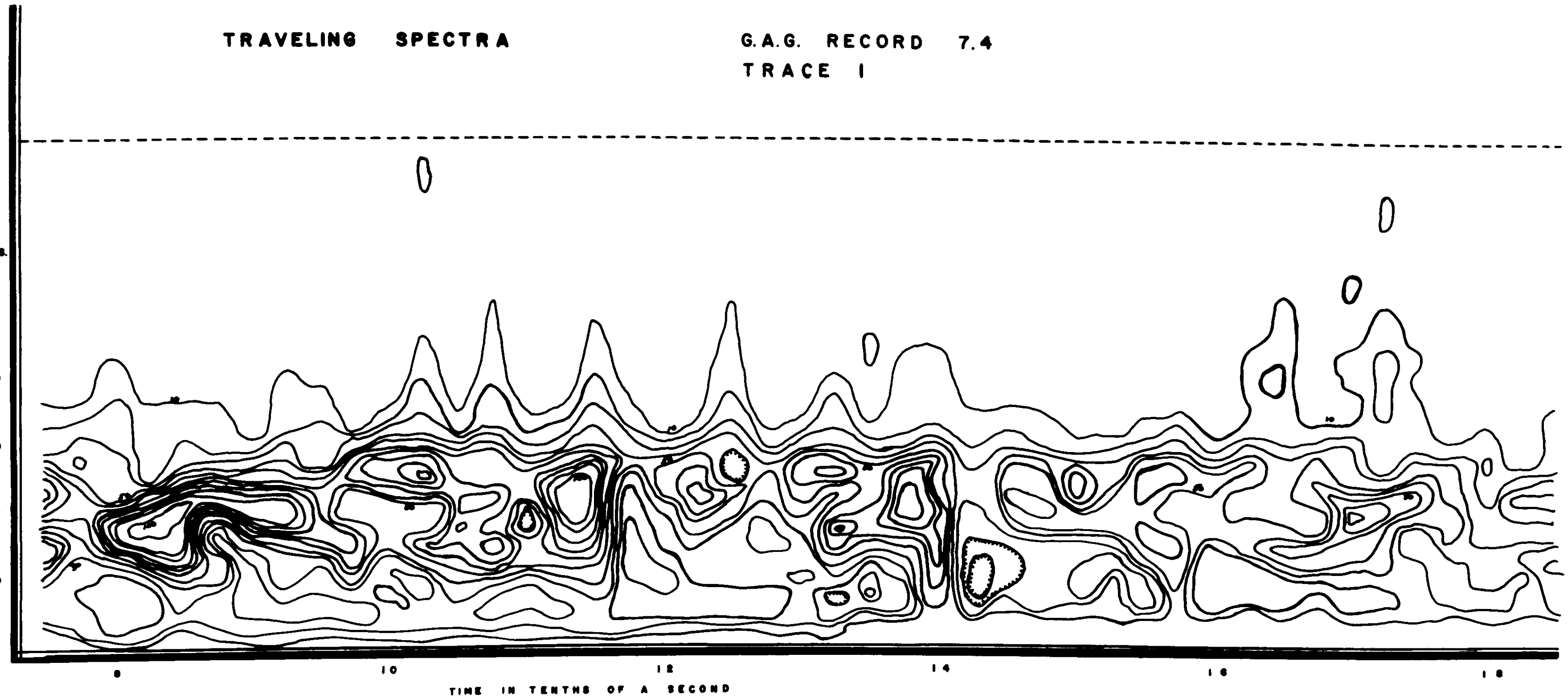

TRAVELING SPECTRA
G.A.G. RECORD 7.4
TRACE I
CPS.
80
60
40
20
0
8
10
12
14
16
18
TIME IN TENTHS OF A SECOND

Fig. (28)

RECORD 7.4
SP 119 - 2300'S
107 - S
Shot Point
Record No. B
Line No. 13
Prospect Elida
Location
Spread
Offset Distance
Charge 70# Depth 142
Time 10:05
Circuit II Gain 54
Low F.C. 35 High F.C. 55
Date 3-16-50
Remarks Wind
SP 120 - 4600'S

TRAVELING SPECTRA GAG RECORD 7.4

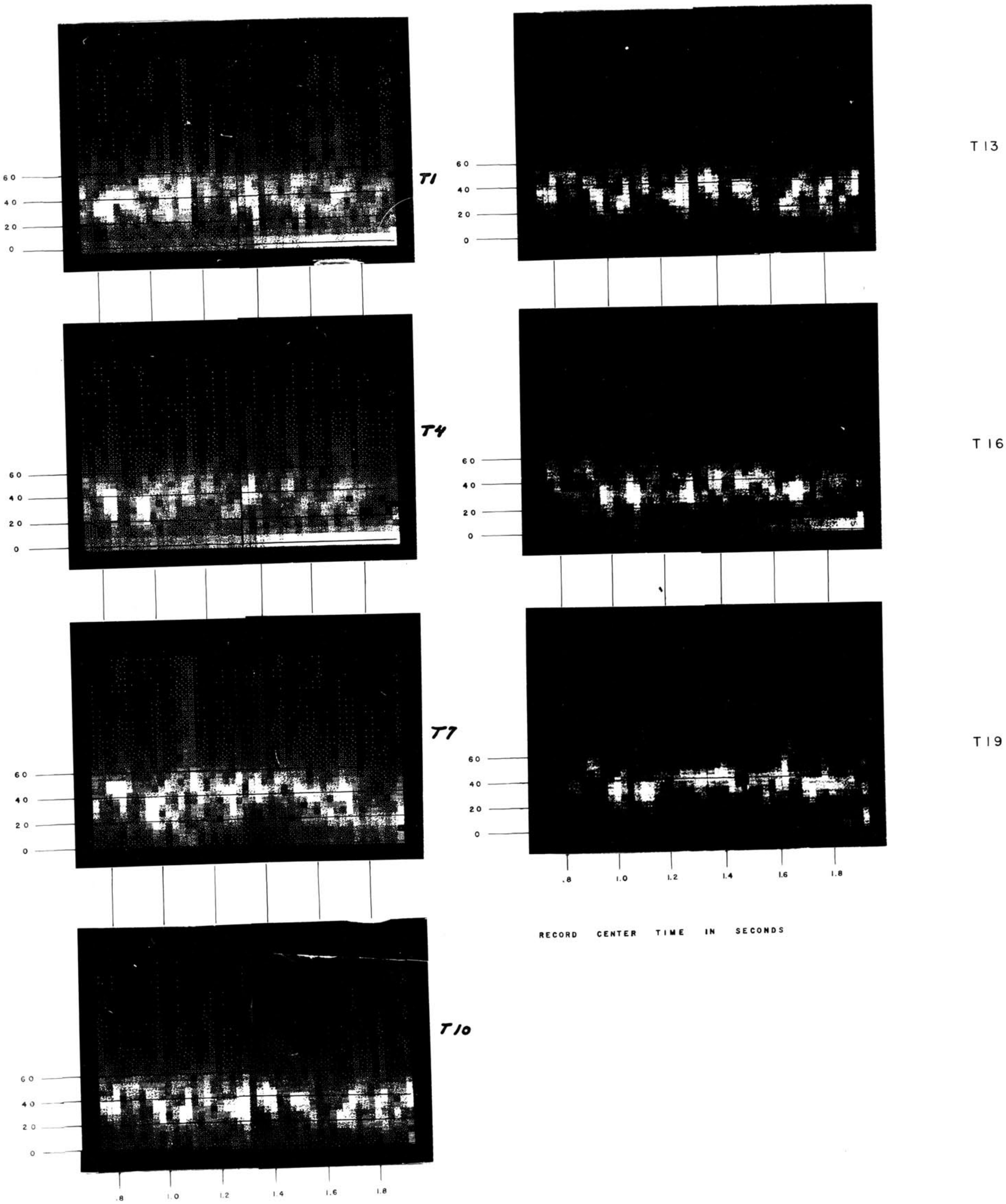


PLATE 13

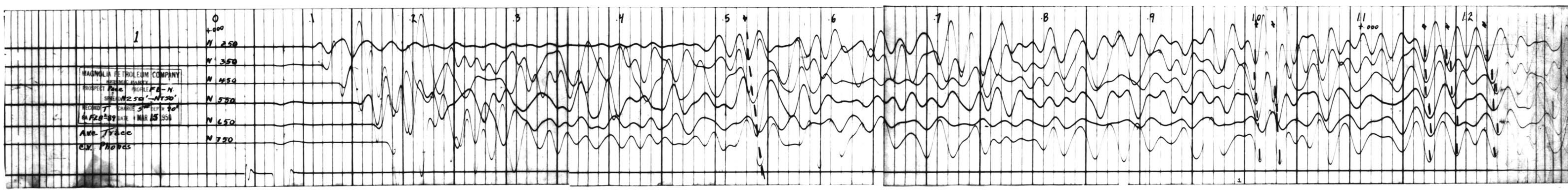

MAGNOLIA PETROLEUM COMPANY
Ave. Trace
N 250
N 350
N 450
N 550
N 650
N 750

Plate (14)

TRAVELING SPECTRA GAG RECORD 101

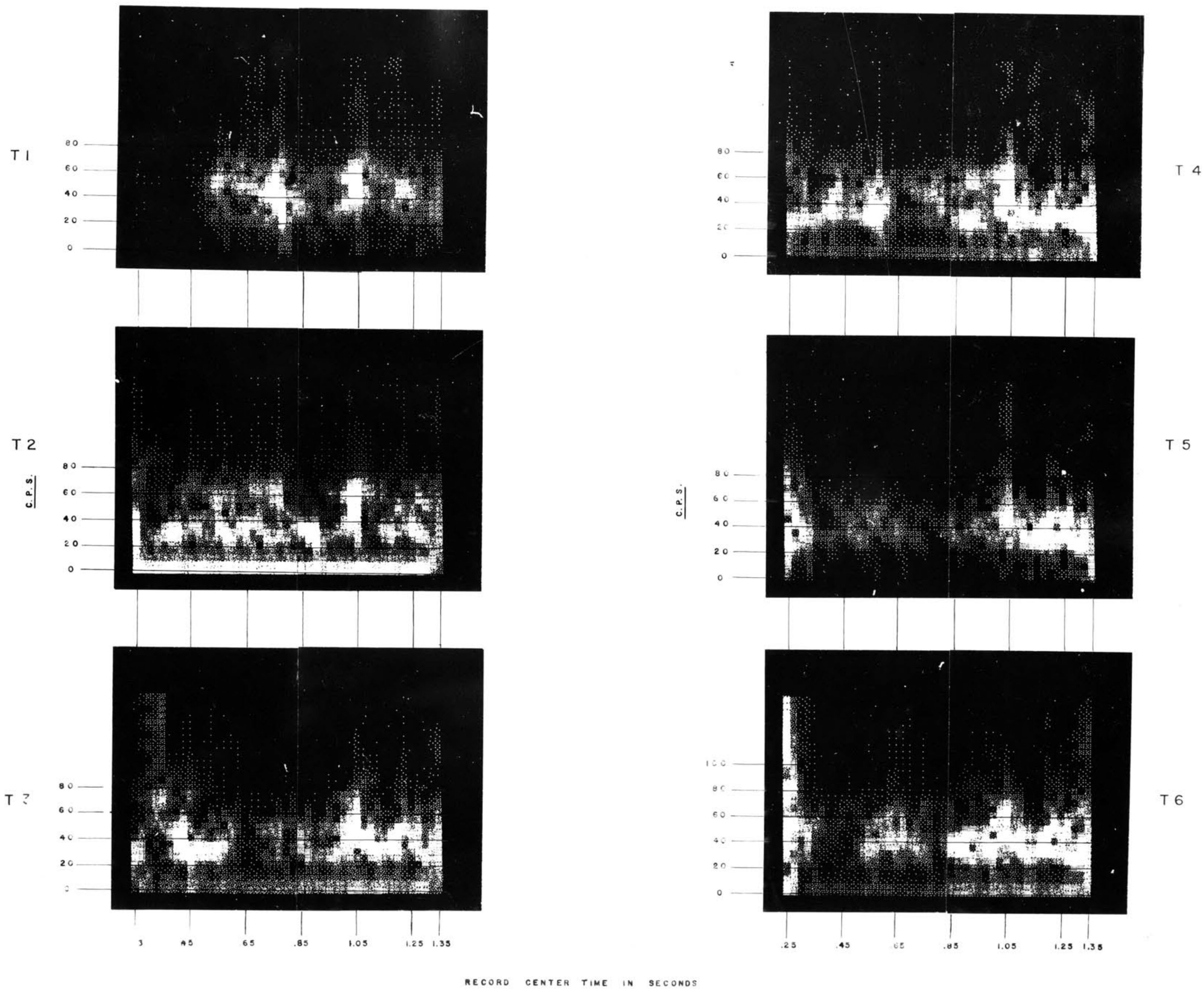


PLATE 15

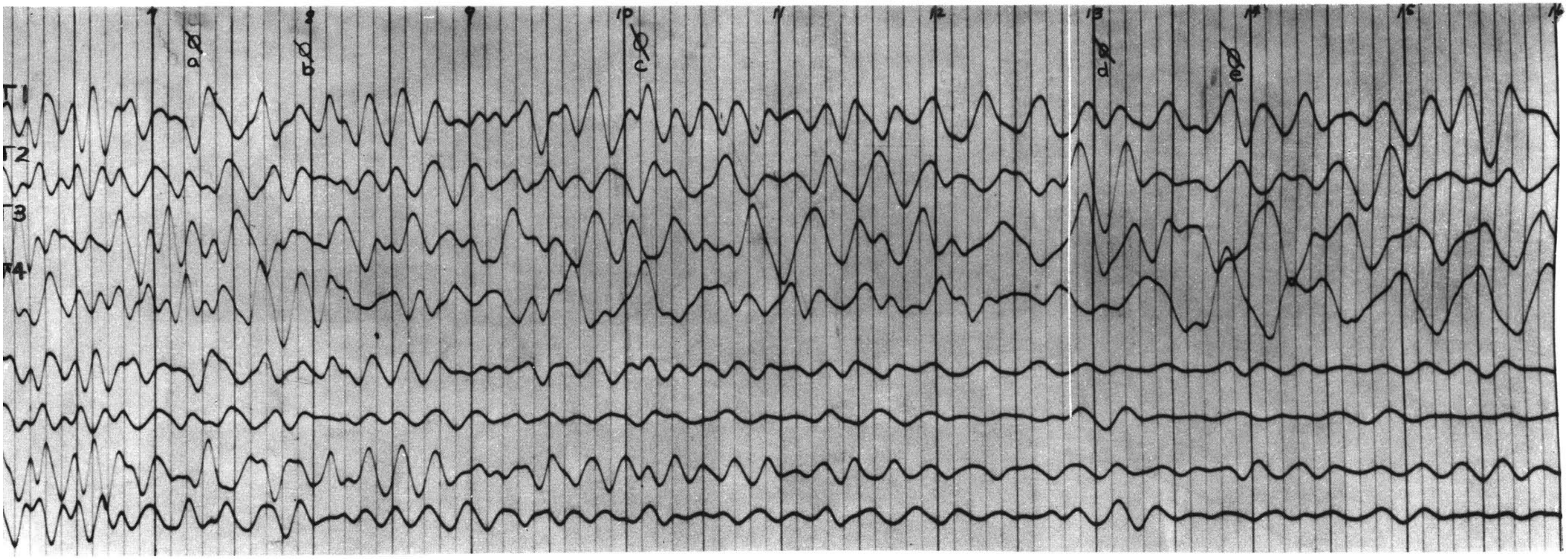


Fig. 1 Record 10.9

PLATE 16

TRAVELING SPECTRA GAC RECORD 109

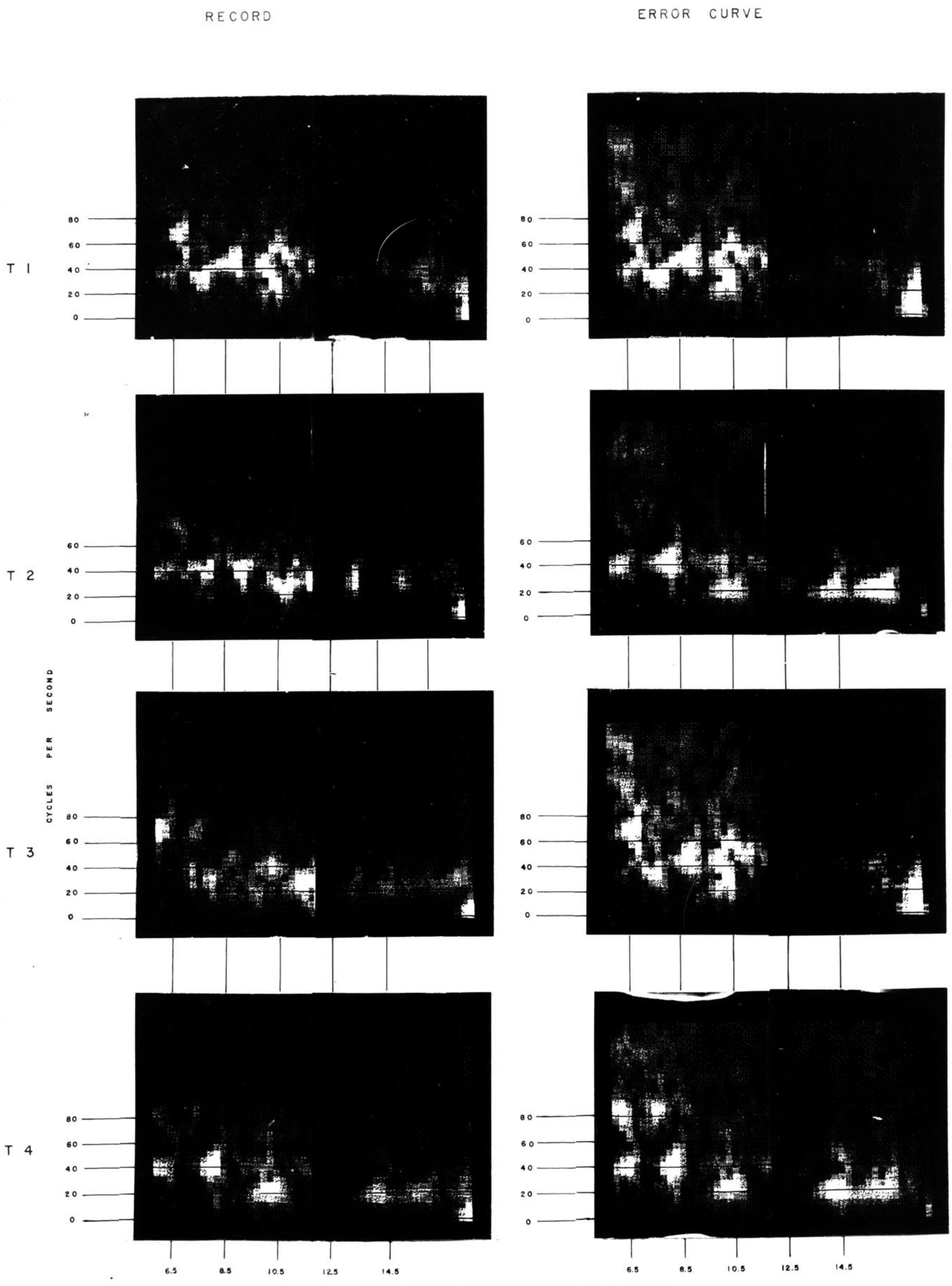


PLATE 17

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BIOGRAPHICAL NOTE

Graduated from Boston College, June 1950 with an A.B. in Physics. In Sept. 1950, entered the Department of Geology and Geophysics, Massachusetts Institute of Technology. Associated with the Geophysical Analysis Group in the capacity of a research assistant from 1952 to 1954.

Appendix A

APPENDIX A

Description and Use of the Power Spectrum Programs

The following outline separates these programs into two groups - the separation being based on the nature of the data which is to be operated on. In each case, whether the output be a spot on a photograph or in digital form, the information is representative of the square of the amplitude spectrum or the power spectrum.

I. A. Input: Auto correlation function

B. Output: An option exists for the use of either direct or delayed printer. In particular there is for the smoothed spectra --

106-43-05018 -	Direct	
106-43-05019 -	Delayed	20 Lags
106-43-05014 -	Direct	
106-43-05015 -	Delayed	40 Lags
106-43-05024 -	Delayed	
106-43-05025 -	Direct	80 Lags
106-43-05016 -	Delayed	
106-43-05017 -	Direct	100 Lags

C. General: Should additional spectrum programs be desired it will be necessary to prepare a table of cosines to be stored in registers 1310 - 1372 and also to present the following registers:

715 -(M-1)
716 -(N-1)
717 +(N-2)
720 -(N-1)
721 -(N-1)
722 -M
731 0.01310
733 -(N-3)
734 -(N-3)
736 +(N-1)
737 -(N-1) /2
740 -(N-1) /2
747 +(N-2)
756 +(N-1) /2
767 A "filing" flexo character if so desired.

where M = no. of spectra to be calculated
N = no. of autocorrelations to be used in the calculation of each spectrum.
e.g. for 100 lags N = 101

Furthermore if an option is desired between a smoothed and unsmoothed spectrum the following registers should be present

263 - sp. 274 ---- if just the smoothed spectrum is desired

1062 - sp. 1071 --- if just the unsmoothed spectrum is desired

The following tapes are then attached in the order indicated below

1) 106-43-05008 (Basic Program)
2) 106-43-05010 (Factoring Routine)
3) a. 106-43-05011 - if direct output is desired
   b. 106-43-05003 - if delayed " " "
4) Tape containing the aforementioned cosine table and preset registers.

D. Preparation of Data: to prevent over-flow it is necessary to factor the data to the form ± . 0XXX e.g. ($+$ 35789 $=$ $+$ .0358)

after the mean has been subtracted. Thereupon the first and last value of those values to be used in the calculation of one spectrum are divided by two.

Having accomplished the foregoing these values ($\frac{R_0}{2}$, R_1, R_2.... $\frac{R_m}{2}$) are placed in registers,

(1373) ----- (1373 $+$ N)

The values of R for the following spectra to be calculated are placed in succeeding registers.

Registers 1373 - to - 3777 are to be used for data storage.

E. Error: All values calculated are in error by approximately $\pm$ 0.3 percent.

All values calculated (L's or U's) should be multiplied by 2x, where x is the reciprocal of the dats scale factor.

F A Typical Performance Request:

1) E S_i 1 "on"

2) 106-43-05018

3) 106-(--)-(-----) Data Tape

4) Put (-M) in Reg. 722 (octal) - (this is done if 722 has not been preset on data tape)

5) S.A. 130

where M is the no. of spectra to be computed.

II. A. Input: Equally spaced trace readings.

B. Output: There are a number of options in this case as is evidenced below. The means of distinction lies in that S.A. request which is starred in F. of this section.

a) Scope or Density Plot Output

OUTPUT	TO BE USED WITH
1. Four pictures 96 Spectra	2-40 lag tapes, 20 pt. skip RS or 2.80 " " 10 " " S.A.3074
2. Two pictures 48 Spectra	1.80 lag tapes 10 pt. skip S.A. 3267
3. Two pictures 48 Spectra	2.80 lag tapes 20 pt. skip S.A. 3261

b) Direct Print Output

1. 40 lag tape	S.A. 3246	Present 3107
2. 80 lag tape	S.A. 3232	

If data for each spectrum desired are on separate tapes, then for:

1. 40 lag tape	S.A. 3212	S.A.3007 for additional calculation
2. 80 lag tape	S.A.3200	S.A.3007 for addational calculation

c) Delayed Print Output

1. 40 lag tape	S.A. 3253	Preset 3107
2. 80 lag tape	S.A. 3242	

If data for each spectrum desired are on separate tapes, then for:

1. 40 lag tape	S.A. 3226	S.A. 3007 for additional calculations
2. 80 lag tape	S.A. 3223	

In each of the above cases the following tapes alone may be used

106-43-05027	40 lag	combined	tape
106-43-05026	80 lag	"	"
106-43-05028	100 lag	"	"

C. General: Should additional combined tapes warrant construction, instructions given in c of the preceding section (I) should be followed in the development of a fundamental tape.

Thereupon the following registers should be preset.

1401	+ N
1402	+ K
1412	- N!
1413	- N!
1414	- N!
1415	- N!
1416	- N!
1424	- M
1425	- M

Where: M = no. of spectra to be calculated from each tape

K = no. of points to be skipped in each succeeding spectrum calculation

N = lag no.

If scope output alone is desired speedier calculation may be obtained by preseting registers --

733 to (-42)
734 to (-42)

The desired tape is then a combination of the following

1. fundamental tape
2. 106-43-05021 = Autocorrelation routine
3. 106-43-05023 = Control routine
4. "Present register" tape.

D. Preparation of Data : All data should be in the form of decimal integers. In general the data tapes should be prepared much in the same way as previous GAG "data" or "trace -reading" tapes, i.e.

1054 - Mean

$$\left.\begin{array}{c} 1055 \\ \downarrow \\ \text{S.A. } 1033 \end{array}\right\} \text{Data}$$

In particular, if density plots or printouts of "traveling spectra" be desired, it would be advisable to prepare tapes containing

a. 511 Successive readings for 40 lag spectra
b. 541 " " " 80 " "
c. 676 " " " 100 " "

If this is accomplished it will be possible to calculate

a. 24 -- 40 lag spectra with 10 pt. skips
b. 24 -- 80 " " " 20 pt. "
c. 24 -- 100 " " " 25 pt. "

E. Error: All digital calculations are about ± 0.3 percent in error.

To obtain the true magnitude of each line spectrum a multiplication factor os 20,000 would be in order.

F. A Typical Performance Request:

a. Scope Output or Print-out of More than One Spectrum

1) E, S_i 1 "on"

2) 106-43-05027
106-43 26 RI
106-43 28

3) Put (106 ---------) (data) in petr

4) a. R.S. -- (if density plot is to be used)
b. S.A. -- (if print-out) -- (if B.)

5) Put (106 ---------) (Next data tape) in petr immediately.
Si L "off"

After computation eases (s:o 3002)
6) E, Si 1 "off" First 30 points of each spectrum
7) 3451 P 12 RI, RI

6) E, S_i 1 "on"
7) 3451 P 12 RI, RI First 41 points of each spectrum
8) 3451 P 15 RI, RS

N.B. 6 - 8 are requested only in the case of scope output.

b. Print Out of One Spectrum at a Time

1) E, S_i 1 "on"

2) 106-43-05022
106-43-05026 RI
106-43-05028

3) Put - (106 -------) (data) in petr.
4) S.A. ------ (of. B.)

If additionsl "one-spectrum" data is had request;

5) Put (106 ------) (nest data tape) in petr.
6) After above computation ceases (S_i 0 130)
S.A, 3007

N.B. The above 4 picture output requests assume that the "phase density" routine in the density plot program (3451 P 12 etc.) has been altered to operate on amplitude spectra. If 3451 P 12 is used and picture 3 and 4 are desired it will be necessary to do the following after pictures 1 and 2 have been obtained.

a) E, S_1 1 off

b) 106-43-050 RI

c) 3451 P 12 RI, RI

Fig. (a)

00		
01		
02		
03		
04		
05		
06		
07		
10		
11		
12		
13		
14		
15		
16		
17		
20		
21		
22		
23		
24		
25		
26		
27		
30		
31		
32		
33		
34		
35		
36		
37		
40	ta 63	
41	ta 70	
42	ca 445	
43	ta 43	
44	sp 114	
45	ta 110	
46	ca 43	
47	md 21	
50	ad 22	
51	td 54	
52	slh 17	
53	ta 43	
54	ca	
55	sr 55	
56	ca 40	
57	sp 46	
60	ca 105	
61	sr 61	
62	ca 42	
63	sp	
64	ca 106	
65	sr 65	
66	ca 111	
67	ta 112	
70	sp	
71	+10	
72	0.00077	
73	1.74000	
74	0.54000	
75	0.36000	
76	0.16000	
77	0.06000	

100	0.46000	
101	0.66000	
102	0.56000	
103	0.06000	
104	1.54000	
105	0.20000	
106	1.22000	
107	+3	
10	ca 0	
11	-9	
12	-9	
13	ca 0	
14	sp 123	
15	ca 121	
16	sr 116	
17	ca 102	
20	sp 45	
21	0.32000	
22	0.20000	
23	ca 122	
24	sp 116	
25	sr 225	(sp 424)
26	ca 740	
27	sp 223	
30	ca 1373	(sp 1433)
31	ta 400	
32	ca 130	Read in
33	ca 131	Data
34	ca 720	
35	sp 130	
36	ca 721	
37	td 720	
40	ca 722	
41	sp 143	
42	sr 0	Stops
43	ca 545	
44	ad 400	
45	ta 545	
46	ca 144	
47	ca 720	
50	sp 143	
51	ca 721	
52	td 720	
53	ca 0	
54	ad 225	
55	ad 231	
56	td 325	
57	ca 727	
60	td 324	
61	ca 727	
62	ca 720	
63	sp 324	
64	ca 721	
65	ta 720	
66	ca 0	
67	ta 724	
70	td 725	
71	ca 723	
72	ca 330	
73	ca 331	
74	ca 230	Reset
75	ta 727	0.00700
76	ca 734	Reset
77	ta 726	+ sign

200	ca 233	
201	sp 153	
202	ca 234	
203	td 233	
204	ca 235	
205	ad 236	
206	td 210	
207	td 213	
10	ca	
11	ad 460	
12	sr 461	
13	ta	
14	ca 211	
15	ca 211	
16	ca 212	
17	ca 212	
20	ca 237	
21	sp 410	
22	sp 145	
23	td 237	
24	ca 744	
25	sr 225	(sp 275)
26	ca 746	S
27	sr 227	
30	sp 133	
31	sr 231	
32	ca 745	
33	sr 433	Reading or film
34	ca 750	Print out
35	sp 237	control
36	sp 253	
37	ca 74	
40	sr 246	
41	ca 237	
42	ca 106	
43	sr 243	
44	sp 243	
45	ca 751	
46	sp 263	→ Printer
47	ca 753	
50	td 237	
51	ca 752	
52	td 751	
53	ca 74	
54	sr 257	
55	sr 755	
56	sp 237	
57	ca 253	
60	ca 255	
61	ta 755	
62	sp 232	
63	ca 755	
64	td 265	
65	ca	Print out
66	sp 40	control
67	ca 265	for ... 's
70	ca 740	
71	sp 265	
72	ca 106	
73	sr 273	
74	sp 1000	
75	md 575	
76	ta 470	V_0
77	ca 743	calculation

300	md 546	
301	ad 460	
302	ta 460	
303	ca 735	
304	ad 736	
305	td 317	
306	ca 766	
307	ad 736	
10	td 320	
11	td 374	
12	td 375	V_m
13	ca 735	calculation
14	ad 717	
15	td 322	
16	ca 741	
17	md	
20	ta	
21	ca 742	
22	md	
23	sp 374	
24	ca 400	
25	md	
26	sp 374	md 316
27	slh 17	
30	ad 546	
31	ta 546	
32	ca 724	
33	ad 723	
34	td 724	
35	ca 756	
36	sp 346	
37	ca 756	Quadrant
40	sp 353	Selection
41	ca 756	
42	sp 361	
43	ca 756	
44	sp 366	
45	sp 325	
46	ad 756	
47	ta 725	
50	ca 757	⊕
51	ta 726	Cosine
52	sp 153	Sign
53	md 760	Routine
54	slh 17	
55	ta 725	⊖
56	ca 761	
57	ta 726	
60	sp 153	
61	ad 756	
62	ta 725	
63	ca 761	⊖
64	ta 726	
65	sp 153	
66	md 760	
67	slh 17	
70	ta 725	⊕
71	ca 757	
72	ta 726	
73	sp 153	
74	ad	
75	ta	
76	sp 1000	
77	+.5000	

NOTES

Power Spectrum

I

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm. F. Webb DATE ______
TAPE NUMBER ______

DL-481

Fig. (b)

Address	Contents	Notes
600		
601		
02		
03		
04		
05		
06		
07		
10		
11		
12		
13		AUTOCORRELATION
14		STORAGE
15		
16		
17		
20		
21		
22		
23		
24		
25		
26		
27		
30		
31		
32		
33		
34		
35		
36		
37		
40		
41		
42		
543		
544		
545		
46		
47		
50		
51		
52		
53		
54		
55		
56		
57		UNSMOOTHED
60		~ L ~
61		SPECTRUM
62		STORAGE
63		
64		
65		
66		
67		
70		
71		
72		
73		
74		
75		
76		
77		

Address	Contents	Notes	
00			
01			
02			
03			
04			
05			
06			
07			
10			
711			
12	ca 730		
13	td 131		
14	sp 130		
15	ca 0	−(M−1)	
16		−(N−1)	TO BE
17		+(N−2)	PRESET
20		−(N−1)	
21		−(N−1)	
22		−M	
23	+1		
24	+0		
25	+0		
26	+1		
27	0 00400		
30	0 00400		
31	0 01310		
32	+1		
33	ca 0	−(N−3)	TO BE
34	ca 0	−(N−3)	PRESET
35	0 00545		
36	ca 0	+(N−1)	TO BE
37	ca 0	−(N−1)/2	PRESET
40	ca 0	−(N−1)/2	
41	+5400		
42	+.4600		
43	+2300		
44	1 62000	UPPER CASE	
45	1 72000	LOWER CASE	
46	0 24000	8	
47	ca 0	+(N−2)	TO BE PRESET
50	−9		
51	−8		
52	−8		
53	0 00073		
54	−9		
55	−9		
56	+10	+(N−1)/2	TO BE PRESET
57	+1		
60	−1		
61	−1		
62	0 00401		
63	0 00546		
64	0 00547		
65	0 01141		
66	0 01140		
67	0.14000	A	
70			
71			
72			
73			
74			
75			
76			
77			

NOTES

Address	Contents	Notes
1000	ca 545	
1001	mr 377	
1002	td 545	
1003	ca 211	[illegible]
1004	mr 377	
1005	td 211	
1006	ca 741	
1007	sp 275	
10		
11		
12		
13	ca 1323	
14	ad 220	
15	td 1323	
16	ad 1013	
17	ad 1015	SUBTRACT
20	ad 720	MO MEAN
21	sp 1013	ROUTINE
22	ca 721	
23	td 720	
24	ad 1012	
25	ad 715	
26	sp 1013	
27	sp 130	
30	ca 741	
31	mr 546	
32	td 1141	
33	ca 743	
34	mr 545	
35	ad 1141	
36	td 1141	
37	ca 743	SMOOTHED
40	mr 547	SPECTRUM
41	ad 1141	$U_1 \to U_{N-2}$
42	td 1141	CALCULATION
43	ad 1031	
44	ad 1032	
45	ad 1034	
46	ad 1035	
47	ad 1036	
50	ad 1040	
51	ad 1041	
52	ad 1042	
53	ad 733	
54	sp 1030	
55	ca 734	
56	td 733	
57	ca 1140	ca 766
60	td 1061	
61	ca	
62	sp 40	(sp 1206)
63	ad 1061	
64	ad 721	
65	sp 1061	
66	ca 106	
67	sc 1067	
70	sc 1070	
71	ca 716	
72	td 220	
73	td 721	
74	ca 722	RESET
75	td 723	CONTROL
76	ca 0	ROUTINE
77	td 545	

	Address	Contents	Notes
	1100	ad 1077	PUT 0's
	1101	ad 720	IN
	1102	sp 1076	545 →
	1103	ca 721	
	1104	td 720	
	1105	ca 735	
	1106	td 1077	
	1107	ca 730	
	10	td 211	
	11	td 144	
	12	ca 762	
	13	td 212	
	14	ca 735	
	15	td 1034	
	16	ca 763	
	17	td 1031	
	20	td 330	
	21	td 331	
	22	ca 764	
	23	td 1040	
	24	ca 765	
	25	td 1032	
	26	td 1035	
	27	td 1036	
	30	td 1042 & td 1041	
	31	sp 212	
	32	ca 767	
	33	sc 1134	
	34	ca 105	
	35	sp 431	
	36	ca 0	
	37		
*	1140		
	41		
	42		
	43		
	44		SMOOTHED
	45		SPECTRUM
	46		—U—
	47		STORAGE
	50		
	51		
	52		
	53		
	54		
	55		
	56		
*	1307		
*	1310		
	61		
	62		
	63		COSINE
	64		TABLE
	65		STORAGE
	66		
	67		
	70		
	71		
	72		
*	1373		
	74		DATA
	75		OR
	76		AUTOCORRELATION
	77		CALCULATION

POWER SPECTRUM (CONT.)

II

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ________ INDEX ________

AUTHOR Wm. P. Walsh DATE ________

TAPE NUMBER ________

DL-481

Appendix B

APPENDIX B

Description and Use of the Amplitude and Cross Spectrum Programs

I. Amplitude Spectrum

A. Input: Discretely spaced points or trace readings.

B . Output: At present options exist for delayed printer and scope (density plot) or scope output only. In each case the spectra attained are representative of spectra of overlapping intervals. We have the following:

1) Scope

106-43-25	Smoothed spectrum	40	"lags"
106-43-06008	Unsmoothed "	40	"
106-43-06009	Smoothed "	80	"
106-43-06016	Unsmoothed "	80	"
106-43-06011	Smoothed "	100	"
106-43-06013	Unsmoothed "	100	"

The 40 "lag" programs have a 10 point skip between intervals and calculate 48 spectr. The density plot routine displays two amplitude spectra pictures - 24 spectra each- and two phase spedtra (corrected) pictures - 24 spectra each.

The 80 "lag" programs, above, have a 20 point skip distance , and calculate 24 spectra. The density plot displays two pictures - one amplitude and one phase spectra (corrected).

The 100 "lag" programs have a 25 point skip between successive intervals, and calculate 24 spectra. The density plot is the same as for the 80 "lag" programs.

We have in addition to the above, for 80 "lags".

106-43-06022 -	smoothed	80 "lags"
106-43-06023 -	unsmoothed	

These programs utilize a 10 point skip between intervals and calculate 48 spectra. The density plot in this case is the same as that mentioned for the 40 "lag" programs.

1) Delayed Printer and Scope

106-43-06014	- Smoothed	40 "lags"
106-43-06015	- Unsmoothed	
106-43-06017	- Smoothed	80 "lags"
106-43-06018	- Unsmoothed	
106-43-06019	- Smoothed	100 "lags"
106-43-06020	- Unsmoothed	

The output for this option will be the "digital" spectra the phase and amplitude spectra of the intervals involved. The print-out is in this order.

We have for

a) 40 "lags" a 10 point skip and a total of 48 spectra
b) 80 " 20 " " " " " " 24 "
c)100 " 25 " " " " " " 24 "

Should, in addition to the above the cosine and / or sine transforms be desired, ca 766 (octal) should be returned to 400 (octal).

In the above description "smoothed" and "unsmoothed" spectrum refers to the cosine and sine transform's being smoothed or not being smoothed by the Tukey method individually, before calculation of the amplitude or magnitude.

Also the term, "lag", is synonomous with "discretely spaced points".

C. General: All of the afore mentioned tapes are combined tapes. The general make-up of each is somewhat as follows:

I Data Factoring Routine

This factors the data and stores same on the magnetic drums, in such a way as to be appropriate for computation of intervals which overlap by 75 percent.

II Phase Amplitude Calculation Routine (3436 M 21, 3436 M 17)

This routine is read in and then thrown on the drums. On the scope output tapes before the "sp block" we have the following

606 sp 615 - circumvent amplitude print out
320 -41
321 -41
514 -40
515 -40
516 -40

460 sp 2251 This must occur before the
467 ca 515 "sp block" in all cases.
473 sp 2067

III Tangent, Angle, Sine, and Sine Sign Table (Tape 3436 M3)

This routine is read in and immediately stored on the drums. Depending on the number of points per interval as new sine table is added on before the "sp block". The values of the sines to be added will be for 0°, $\frac{180^\circ}{n}$, $2(\frac{180^\circ}{n})$... where n equals the number of points per interval. The number of points per interval is not to exceed 100.

IV Sine and Cosine Transform Calulation Routine
1 (Tape 3436 M20)

2 Data Read in Routine and Phase Correction Routine (Tape 106-43-18)

A cosine table (the reverse of the above sine values) is added. The first value being placed in 1000 (octal).

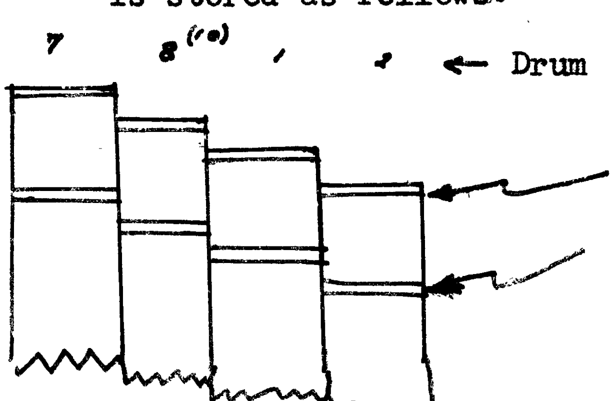
On the scope output tapes the following modifications occur before the sp block

733	-45
734	-45
2067	sp 2070
2100	sp 2101
2127	sp 2130
2166	sp 2176
2423	sp 473
2110	ad 2472
2137	ad 2472
24.72	+ 41 (decimal)

In addition to the above changes we have for the 80 and 100 lag tapes,

2213	sp 2220	
2223	-9	After third sp block
2415	-14	
627	-9	After first sp block

Registers 560 through 563 contain drum and register of the first factored (halved) value of the overlap in question. In explanation, four drum groups are used for the factored data each having appropriate and different values factored by .5. Diagramatically, the data for each spectrum is stored as follows:



D. Preparation of Data: Data is prepared much in the same fashion as other GAG data. All points or trace readings are positive decimal integers and appear in registers 1055 etc. The mean appears in 1054.

In particular we have for

40 lag spectra	and	a	total	of	48	spectra	-	511	points
80 " "	"	"	"	"	24	"	-	541	"
100 " "	"	"	"	"	24	"	-	676	"

All data is typed for 5-56 bisic conversion. A "start at 0" is usual for most data tapes. Howecer, "start at 1033" may appear if this data is to be used with the auto-correlation spectrum programs.

E. Typical Performance Requests

1) Scope Output - (40 lags)

1) E. S_i 1 "off"

A
2) fb 106-43 -----Data - RI
3) fb 106-43 -25 RI. RI, RI, RI
-- After computation classes - (Si 0 2153)

B
4) E, Si 1 "off"
5) 3451 P 12 RI, RI

C
4) E, Si 1 "on"
5) 3451 P 12 RI, RI
6) 3451 P 16 RI, RS

B and C are the density plotting routines

B - is used should the first 31 points of each spectrum be desired

C - is used if the first 41 points of each spectrum are desired.

In this connection, we may reiterate that points of the spectrum are plotted vertically while center time or succeeding spectra are plotted horozontally.

2.. Delayed Print Output

This is the same as the foregoing save that B or C are omitted.

II.. Cross Spectrum

A.. Input: Discrete points of the crosscorrelation function.

B.. Output: Cross spectrum and phase difference. At present only delayed print output is possible; the tapes used in calculating amplitude spectra are applicable in this section

C.. Preparation of Data: Bothe and should have the mean subtracted and also befactored by .0010. They both should appear in registers 1055 ---- etc. All data for succeeding spectra should follow in succeeding registers. After all the data has been prepared in will be necessary to make up a small routine. In particular,

a) for S.A. 40

```
40  ca 45
41  si 707
42  ca 46
43  bo 1055
44  si 0
45  0.34100
46  + N  (the quantity of data)
```

b) for S.A. 40

```
40  ca 45
41  si 707
42  ca 46
43  bo 1055
44  si 0
45  0.41000
46  + N  (the quantity of data)
```

The above should be attached to each tape.

Instead of the aforementioned it is possible to prepare the cross-correlations as other GAG data tapes - mean in 1054, first number in 1055 etc. in decimal integers.

If the above is the alternative the following presetting of registers must occur, on each tape (one for Φ_{MN} and one for Φ_{NM}).

```
 74 - (N-1)
 77 +  A
100 - (M-1)
102 -  M                          and, S. A.   0
104 +  N
105 0.34100 - for Φ NM
    0.41000 - for Φ NM
```

WHERE : N = no. of data points

$M = \frac{N}{A} + 1$

A = + 40; +80; + 100 (depending on what lag spectra is desired)

These tapes are then to be used in conjunction with the cross-correlation factoring routine - 106-43-06023.

D. Typical Performance Requests

E Si 1 "off"

106 - 43 - (Φ_{MN} data) RI

106 - 43 - (Φ_{NM} data) RI

Si 1 "on"

-- Starting at second block-

106-43-(XXXXX) RI, RS; RI, RS; RI

106-43-05030 RI, RS

The above supposes that the data has been factored and prepared as mentioned in the foregoing section. If it has not, and it appears as decimal integers then the following request would be appropriate.

E, Si 1 "off"

106-43- (ϕ_{MN} data) RI

106-43- 06023 RI

106-43 - (ϕ_{NM} Data) RI

106-43- 06023 RI

si 1 "On"

Starting at second block

106-43-(06xxx) RI,RS; RI, RS; RI

106-43-05030 RI, RS.

Address	Instruction	Remarks
00		
01		
02		
03		
04		
05		
06		
07		
10		
11		
12		
13		
14		
15		
16		
17		
20		
21		
22		
23		
24		
25		
26		
27		
30		
31		
32		
33		
34		
35		
36		
37		
40	ta 63	
41	ta 20	
42	sp 225	or sp 176
43	to 113	
44	sp 114	
45	to 110	
46	cm 113	
47	mh 21	
50	ad 72	
51	td 54	~ PRINT ~
52	sl 17	~ OUT ~
53	to 113	
54	ca	USING
55	sc 55	DIRECT
56	ad 110	OR
57	cp 46	DELAYED
60	ca 105	PRINTER
61	sc 61	
62	ad 112	
63	cp	
64	ca 106	
65	sc 65	
66	ca 111	
67	to 112	
70	sp	
71	+10	
72	0.00073	
73	1.74000	
74	0.52000	
75	0.36000	
76	0.16000	
77	0.26000	

Address	Instruction	Remarks
100	0.46000	
101	0.66000	
102	0.56000	
103	0.06000	
104	1.54000	
105	0.20000	
06	1.22000	
07	+3	
10	ca 0	
11	-9	
12	-9	
13	sp 0	
14	cp 123	
15	ca 121	
16	sc 116	
17	ca 107	
20	sp 45	
21	0.32000	
22	0.72000	
23	ca 122	
24	cp 116	
25	ca 0	
26	sr 0	
27	sp 0	
30	ca 400	
31	ad 1412	
32	to 1100	
33	ad 131	M_0 Calculation
34	ad 720	
35	cp 130	
36	ca 721	
37	to 720	
40	ca 1100	
41	mh 377	
42	to 1100	
43	ca 0	
44	ad 725	
45	ad 731	
46	td 325	Control
47	ca 727	and
50	td 324	Reset
51	ad 727	Routine
52	ad 720	For
53	cp 324	Calculation
54	ad 721	of
55	to 720	$M_1 \to M_{n-2}$
56	ca 0	
57	to 724	
60	to 725	
61	ad 723	
62	ad 330	
63	ad 331	
64	ca 730	
65	to 727	
66	ca 732	
67	to 726	
70	ad 733	
71	sp 143	
72	ca 734	
73	to 733	
74	ad 735	
75	ad 736	
76	td 200	
77	td 205	

NOTES

Address	Instruction	Remarks
200	td 216	
201	td 220	
202	ca	Calculation
203	ad 1412	of
204	su 1413	M_{n-1}
205	to	
06	ad 203	
07	ad 203	
10	ad 204	
11	ad 204	
12	ad 737	
13	cp 202	
14	ca 740	
15	to 737	
16	ca	
17	mr 322	
20	to	
21	sp 222	
22	td 223	Control
23	ca	Routine
24	sp 40	For
25	ad 223	Printing
26	ad 220	M's
27	cp 223	
30	ca 106	
31	sc 231	
32	ca 741	
33	mr 1100	Calculation
34	to 1245	of
35	ca 742	V_0
36	mr 1101	
37	ad 1245	
40	to 1245	
41	ca 735	
42	ad 736	Calculation
43	td 255	of
44	ca 766	V_{n-1}
45	ad 736	
46	td 256	
47	td 261	
50	td 262	
51	ca 735	
52	ad 217	
53	td 260	
54	ca 741	
55	mr	
56	to	
57	ca 742	
60	mr	
61	ad	
62	to	
63	sp 270	
64	0.01413	
65	su 0	
66	su 0	
67	su 0	
70	ca 741	
71	mr 1101	Calculation
72	to 1246	of
73	ca 743	V_1
74	mr 1100	$\downarrow$
75	ad 1246	
76	to 1246	V_{n-2}
77	ca 743	

Address	Instruction	Remarks
300	mr 402	
301	ad 1246	
302	to 1246	
303	ad 271	
304	ad 272	
305	ad 274	
06	ad 275	
07	ad 276	
10	ad 300	
11	ad 301	
12	ad 302	
13	ad 733	
14	cp 270	
15	ca 734	
16	to 733	
17	sp 400	→ PRINT V's
20	ca 721	
21	to 720	
22	sp 400	→ PRINT M's
23	su 0	
24	ca	Calculation
25	mr	of
26	mh 726	M_1
27	sl 17	$\downarrow$
30	ad 1101	
31	to 1101	M_{n-2}
32	ca 724	
33	ad 723	
34	to 724	
35	su 756	
36	cp 346	Sign
37	su 757	Control
40	cp 352	Routine
41	su 756	
42	cp 361	
43	su 756	
44	cp 366	
45	sp 335	
46	ad 756	
47	to 725	
50	ca 757	+
51	to 726	
52	sp 143	
53	mh 760	
54	sl 17	Cosine
55	to 725	− Sign
56	ca 761	Routine
57	to 736	
60	sp 143	
61	ad 756	
62	to 725	−
63	ca 761	
64	to 726	
65	sp 143	
66	mh 760	
67	sl 17	
70	to 725	+
71	ca 757	
72	to 726	
73	sp 143	
74	0.4012	
75	+352	
76	ca 0	
77	+.5000	

Amplitude & Phase Spectrum

I

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______

AUTHOR Wm P Walsh DATE ______

TAPE NUMBER ______

Fig. (c)

DL-481

Fig. (d)

Address	Instruction	Remarks
400	ca 266	CONTROL
401	td 402	ROUTINE
402	ca	FOR
403	sp 410	PRINTING
404	ad 402	V's
405	ad 721	
06	sp 402	
07	ca 106	
10	rc 410	
11	ad 716	
12	ts 720	RESET TO -(N-1)
13	ts 721	
14	ca 732	RESET TO +1
15	ts 723	
16	ca 1245	
17	ts 1412	
20	ad 416	TRANSFER V's
21	ad 417	TO → 1412 ↓ 556
22	ad 720	
23	sp 416	
24	ca 0	
25	ts 1100	
26	ad 425	
27	ad 714	~ RESET ~
30	sp 424	
31	sp 1776	~ ROUTINE ~
32	td 425	0.00100
33	ca 617	0.01412
34	td 131	
35	td 203	
36	td 727	
37	sp 2162	
40	td 204	0.01413
41	ca 735	0.0490
42	td 274	
43	ca 263	0.01101
44	td 330	
45	td 331	
46	td 271	
47	ca 764	0.0403
50	td 300	
51	ca 765	0.01246
52	td 272	
53	td 275	
54	td 276	
55	td 301	
56	td 302	
57	sp 1600	→ READ OUT & IN ROUTINE
60	ca 1100	
61	mr 770	
62	ts 400	
63	su 460	
64	ad 462	SUBTRACT
65	ad 720	
66	sp 460	MEAN
67	ca 1245	
70	mr 721	
71	ts 1245	
72	ad 467	
73	ad 471	
74	ad 721	
75	sp 467	
76	ad 716	
77	ts 720	

Address	Instruction	Remarks
500	ts 721	
501	ca 735	
502	td 460	
503	td 462	
504	ca 713	
505	td 467	
06	td 471	
07	ca 735	
10	td 511	
11	ca	
12	ad 1245	
13	ts 1412	
14	ad 511	
15	ad 512	
16	ad 513	
17	ad 720	ADD & SUBTRACT
20	sp 511	
21	ad 716	SUM & DIFF
22	ts 720	
23	ca 212	
24	ts 512	
25	ad 760	
26	sp 2040	
27	ca 761	
30	ts 760	
31	ca 711	
32	ts 512	
33	ca 730	
34	td 513	
35	sp 2011	
36	mr 0	
37	mr 0	
40	mr 225	
41	ad 744	
42	rc 542	
43	ca 746	NUMBERING
44	rc 544	(FILING)
45	ca 745	ROUTINE
46	rc 546	
47	ca 105	
50	rc 550	
51	ca	
52	rc 552	
53	ca	
54	rc 554	
55	sp 561	
56	mr 0	
57	mr 0	
60	ad 551	
61	ad 553	
62	ca 106	
63	rc 563	
64	sp 460	→ SUBTRACT MEAN
65	-201	
66	mr 0	
67	ca 703	
70	mr 707	READ OUT
71	ca 652	SINE SIGN
72	ts 346	
73	ca 701	
74	mr 703	READ IN
75	ca 652	COSINE SIGN
76	ts 346	
77	sp 656	

NOTES

Address	Instruction	Remarks
600	ca 701	
601	mr 707	READ OUT COSINE
602	ca 652	SIGN
603	ts 346	
604	ca 702	→ sp 2043
605	ts 727	0.01557
06	ts 230	
07	td 131	
10	td 417	
11	td 203	
12	ca 615	0.01560
13	td 204	
14	sp 620	
15	0.01560	
16	+327	
17	0.01412	
20	ca 703	
21	mr 703	R.I. SINE SIGN
22	ca 652	
23	ts 346	
24	ca 704	
25	mr 707	R.O COSINES
26	ca 707	
27	ts 1000	
30	ca 705	
31	mr 703	R.I. SINES
32	ca 707	
33	ts 1000	
34	ca 706	
35	td 457	
36	sp 1063	→ CALCULATE N's
37	ca 617	
40	ts 727	0.01412
41	ts 730	
42	td 131	
43	td 417	RESET
44	td 203	ROUTINE
45	ca 264	0.01413
46	td 204	
47	ca 655	0.00600
50	td 457	
51	sp 567	
52	+322	
53	0.00400	
54	0.01066	
55	0.00600	
56	ca 705	
57	mr 707	R.O. SINES
60	ca 707	→ sp 666
61	ts 1000	
62	ca 704	
63	mr 703	R.I. COSINES
64	ca 707	
65	ts 1000	
66	ca 653	
67	td 317	
70	ca 374	
71	mr 707	R.O. 130-667
72	ca 375	
73	ts 130	
74	ca 1077	
75	mr 703	R.I PHASE
76	ca 1756	PROGRAM
77	ts 125	

Amplitude & Cross Spectrum

II

Address	Instruction	Remarks
700	sp 1757	
701	0.20346	
702	0.01557	
703	0.30346	
704	0.21000	
705	0.31000	
06	0.00637	
07	+51	
10	0.02130	
11	ad 1245	
12	mr 1245	
13	0.01245	
14	mr 0	-201 -(2N-1)
15	mr 0	-(M-1)
16	mr 0	-(N-1) TO BE
17	mr 0	+(N-2) PRESET
20	mr 0	-(N-1)
21	mr 0	-(N-1)
22	mr 0	-M
23	+1	
24	+0	
25	+0	
26	+1	
27	0.01412	
30	0.01412	
31	0.01000	
32	+1	
33	-98	-(N-3) TO BE
34	-98	-(N-3) PRESET
35	0.01100	
36	+100	+(N-1) TO
37	-50	-(N-1)/2 BE
40	-50	-(N-1)/2 PRESET
41	+.5400	
42	+4600	
43	+.2300	
44	1.62000	FLEXO CHARACTER
45	172000	
46	mr 0	2, C, ETC TO BE
47	+99	+(N-2) PRESET
50	-9	
51	-8	
52	-8	
53	0.00073	
54	-9	
55	-9	
56	+50	
57	+1	
60	-1	
61	-1	
62	0.01413	
63	0.01101	
64	0.0402	
65	0.01246	
66	0.01245	
67	0.14000	A
70	mr 0	
71	mr 0	
72	mr 0	
73	mr 0	
74	mr 0	
75	mr 0	
76	mr 0	
77	mr 0	

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm P Walsh DATE ______
TAPE NUMBER ______

DL-481

Fig. (e)

1000	+.9999	
1001	+.9995	
1002	+.9980	
1003	+.9956	
1004	.9921	
1005	.9877	
06	.9823	
07	.9759	
10	.9686	COSINE
11	.9603	
12	.9511	VALUES
13	.9409	
14	.9298	USED
15	.9177	
16	.9048	FOR
17	.8910	
20	.8763	100 LAGS
21	.8607	
22	.8443	
23	.8271	
24	.8090	
25	.7902	COSINE & SINE
26	.7705	STORAGE
27	.7501	
30	.7290	
31	.7071	
32	.6845	
33	.6613	
34	.6374	
35	.6129	
36	.5878	
37	.5621	
40	.5358	
41	.5090	
42	.4818	
43	.4540	
44	.4258	
45	.3971	
46	.3681	
47	.3387	
50	.3090	
51	.2790	
52	.2487	
53	.2181	
54	.1874	
55	.1564	
56	.1253	
57	.0941	
60	.0628	
61	.0314	
62	+0	//////////
63	ca 654	
64	td 317	
65	sp 130	
66	ca 1245	
67	mh 200	
70	sl 17	
71	ts 1245	
72	ca 1066	
73	ca 1021	
74	ca 720	
75	[illegible]	
76	sp 200	
77	ts 1000	

1100		
1101		
02		
03		M & N STORAGE
04		
05		
06		
07		
10		
1411		//////////////
1412		
13		
14		J & V STORAGE
15		
1516		//////////////
1517		
20		
21		
22		II & N STORAGE
1723		//////////////
24	ca 0	
25	0.21000	
26	+346	
27	+5	
30	ca 1725	
31	ad 1726	
32	ts 1724	M (mod)
33	ca 1724	
34	mr 707	
35	ca 1727	
36	ts 651	
37	ca 1744	
40	mr 703	
41	ca 1745	
42	ts 130	
43	sp 1766	M (mod)
44	0.40130	
45	+424	
46	0.3400	
47	+202	//////////
50	ca 1746	
51	mr 707	
52	ca 1747	
53	ts 400	
54	ca 624	
55	sp 631	
56	+355	
57	ca 1765	
60	mr 707	R.O. 1063-1077
61	ca 1764	
62	ts 1063	
63	sp 500	
64	+13	
65	0.31063	
66	ca 1765	
67	mr 703	R.I. 1063-1077
70	ca 1764	
71	ts 1063	M (mod)
72	ca 713	sp 2154 →
73	td 1066	
74	td 1071	
75	sp 540 →	
76	ca 565	
77	ts 714	

NOTES

2000	ca 735	
2001	sp 432	
2002	ca 0	
2003	ca 0	
2004	ca 0	
2005	mh 124	
06	sl 17	M (mod)
07	ts 170	
10	sp 230 →	
11	ca 0	
12	ts 1100	
13	ca 2012	PUT 0 IN M & N
14	ca 714	STORAGE
15	sp 2011	
16	ca 565	
17	ts 714	
20	ca 735	
21	td 2012	
22	sp 130	→ TO CALCULATION OF M & N
23	ca 1100	
24	ad 1000	
25	ts 1100	
26	ca 735	
27	sp 2022 →	//////////
30	ca 125	
31	sp 2034	M (mod)
32	ca 122	
33	sp 242	
34	mh 124	
35	sl 17	
36	ts 125	
37	sp 2032	
40	ca 702	
41	td 513	
42	sp 507 →	
43	ca 766	
44	td 416	
45	ca 702	
46	sp 605 →	
47	ca 766	
50	td 416	
51	ca 617	
52	sp 640 →	//////////////
53	sl 126	
54	ca 2060	
55	sc 2055	
56	sl 134	
57	ca 0	300
60	1.42000	
61		
62		
63		//////////
64	1.42000	
65	0.34100	
66	0.14100	
67	sl 134	④
70	ca 111	
71	ts 112	M (mod)
72	ca 515	
73	ts 514	②
74	ca 733	sp 2103
75	sp 2020	ca 516
76	ca 2024	ts 514
77	ca 2030	ts 515

AMPLITUDE & CROSS SPECTRUM (ABRIDGED)

III

	①	②	
2100	ca 2122	sl 134	④
2101	ca 514	ca 111	
2102	sp 2024	sp 2121	
2103	ca 2125		
2104	mr 707		B.O. PHASE
2105	ca 626		FOR DENSITY
06	ts 733		PLOT
07	ca 2125		
10	ad 626	(ad 2456)	
11	ts 2125		
12	ca 256	sp 2116	
13	td 2074		
14	td 2120		
15	td 2122		
16	ca 515		
17	sp 540	→ sp 520	
20	ca 733		
21	ad 2126		
22	ts 733		Mod ⑦
23	sp 2026		
24	0.34100		AMPLITUDE DRUM
25	0.14100		PHASE "
26	+.3600		
27	sl 134	④	
30	ca 111	sp 2075	
31	ts 112		
32	ca 2124		
33	mr 707		B.O. AMPLITUDE
34	ca 626		FOR DENSITY
35	ts 733		PLOT
36	ca 2124	sp 2212	
37	ad 626	(ad 2456)	
40	ts 2124		
41	ca 516		
42	sp 623		
43	ca 2065	sp 2405	
44	ts 2124		
45	ca 2066		
46	ts 2125		
47	sl 126		
50	ca 2064		
51	sc 2151		
52	sl 134		//////////
53	ca 0		STOPS HERE!
54	ca 704		R.I. COSINES
55	ad 703		
56	ca 707		
57	ts 1000		
60			
61			
62	ca 216		
63	ts 220		RESET COUNTER
64	ca 762		
65	sp 440		//////////
66	ca 550		
67	sp 2177		
70	sl 170		
71	ca 106		
72			
73	CONTINUED		
74			
75			
76			
77			

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR ______ DATE ______
TAPE NUMBER ______

DL-481

Fig. (I)

00		
01		
02		
03		
04		
05		
06		
07		
10		
11		
12		
13		
14		
15		
16		
17		
20		
21		
22		
23		
24		
125	ca 0	
126	+0	
127	-0	
130	ca 140	
131	cp 135	→ Re ⊖
132	ad 147	Re ⊕ ↓
33	cp 326	QUADRANT SEARCH CONTROL ROUTINE
34	cp 142	
35	ca 147	
36	mr 1412	
37	cp 326	|Re| = 0.
40	cp 142	
41	0.01100	
42	ca 1557	
43	cp 271	→ Im ⊖
44	ca 1412	Im ⊕ ↓
45	cp 322	
46	mr 1557	
47	cp 223	→ |Im| > |Re|
50	ca 1557	
51	dv 1412	
52	sl 17	CALCULATION OF TANGENT
53	ts 125	
54	ca 1412	
55	cp 206	→ Re ⊖
56	ca 1100	① Re +
57	mr 125	② Im +
60	cp 203	③ |Re| > |Im|
61	ca 172	
62	ad 173	CHOOSE PROPER ANGLE CORRESPONDING TO CALCULATED TANGENT
63	td 164	
64	ca	
65	ts 233	
66	sp 200	
67	0.01557	
70	sl 0	
71	0.01412	
72	0.01245	
73	+0	
74	-1	
75	+1800	
76	+.0900	
77	0.00244	

NOTES

DL-481

200	ca 126	
201	ts 173	
202	sp	⇑
203	ts 156	
204	ca 123	
205	sp 156	
206	ca 1557	
207	dv 1412	
10	sl 17	
11	mh 174	CALCULATION OF TANGENT
12	sl 17	
13	ts 125	
14	ca 254	
15	td 202	① Re −
16	sp 156	② Im +
17	ca 125	③ |Re| > |Im|
20	su 233	
21	ts 233	
22	sp	⇑
23	ca 1412	→ Im ⊕ > Re
24	cp 257	→ Re ⊖
25	ca 1412	Re ⊕ ↓
26	su 1557	
27	ts 170	CALCULATION OF TANGENT
30	ca 126	
31	su 170	
32	su 126	
33	cp 246	→ |Re| = |Im|
34	cp 1412	↓ Im > Re
35	ca 1412	
36	dv 1557	
37	sl 17	
40	ts 125	
41	ca 177	
42	td 202	
43	sp 156	
44	ca 126	
45	sp 220	
46	ca 137	
47	ts 233	Re = Im
50	sp	⇑
51	0.00233	
52	0.00350	
53	0.00311	
54	0.00217	
55	0.00265	
56	0.00233	
57	ca 255	
60	td 250	
61	td 202	
62	ca 1412	
63	ad 1557	
64	sp 2005	
65	ca 125	
66	su 233	
67	ts 233	
70	sp	⇑
71	ca 137	
72	su 1557	
73	cp 340	→ Im = −0
74	sp 277	→ Im = +0
75	ts 233	
76	sp	⇑
77	ca 1557	

① PHASE OR PHASE DIFFERENCE CALCULATION

② MAGNITUDE CALCULATION

I

300	mh 174	
301	sl 17	
302	ts 1557	
303	ca 253	
304	td 202	
305	td 222	
306	td 250	
07	td 270	
10	sp 144	
11	ca 233	
12	mh 174	
13	sl 17	
14	ts 233	
15	sp	⇑
16	0.01412	
17	0.01557	
20	−101	−(xx) } TO BE PRESET
21	−101	−(xx)
22	mh 174	
23	sl 17	
24	sp 146	
25	si 0	
26	ca 176	
27	ts 233	
30	ca 1557	
31	cp 333	
32	sp	⇑
33	ca 233	
34	mh 174	
35	sl 17	
36	ts 233	
37	sp	⇑
40	ca 1412	
41	cp 344	
42	ca 245	
43	sp 275	
44	ca 125	
45	ts 233	
46	sp	⇑
47	si 0	
50	ca 321	← START OF PHASE CALCULATION
51	cp 353	
52	sp 450	→ PRINT OUT
53	ca 252	
54	td 270	
55	td 276	REGISTER PRESETTING
56	td 315	
57	td 332	
60	td 337	
61	td 346	
62	td 202	
63	td 222	
64	td 250	
65	ca 251	
66	td 165	
67	td 220	
70	td 221	
71	td 247	
72	td 266	
73	td 267	
74	td 275	
75	td 314	
76	td 314	
77	td 317	

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______

AUTHOR Wm P Wohl DATE ______

TAPE NUMBER ______

Fig. (8)

400	td 333	
401	td 336	
402	td 345	
403	ca 121	Resume
404	td 130	
405	td 136	
406	td 144	
07	td 151	
10	td 154	
11	td 207	Register
12	td 223	Preset
13	td 225	
14	td 235	
15	td 262	
16	td 340	
17	ca 167	Re adjust
20	td 142	
21	td 146	
22	td 153	
23	td 206	
24	td 226	
25	td 236	
26	td 263	
27	td 270	
30	td 277	
31	td 302	
32	td 330	
33	ca 141	Re 1100
34	td 156	
35	ao 162	
36	ao 121	
37	ao 251	
40	sp 130	
41	[illegible] 0	
42	0.31100	
43	+202	
44	0.40620	
45	+22	
46	0.21100	
47	+340	
50	ao 256	
51	ts 251	
52	ca 316	Reset
53	ts 171	
54	ca 317	
55	ts 167	
56	ca 320	
57	ts 321	
60	ca 251	(sp 2251)
61	td 462	
62	ca	Phase
63	sp 40	Print-out
64	ao 462	Control
65	ao 514	
66	cp 462	
67	ca 515	Reset
70	ts 514	sp 2117
71	ca 106	
72	rc 472	
73	sp 2067	
74	sp 2300	
75	[illegible] 0	
76	[illegible] 0	x_n - - - and $\sqrt{a}$
77	[illegible] 0	$\frac{1}{2}\left[\frac{a}{x_n} - x_n\right]$

NOTES

500	ca 444	
501	[illegible] 707	R.O. 670-777
502	ca 445	
503	ts 670	
504	ca 442	
505	[illegible] 703	R.I. Tangents and
506	ca 443	Angles
07	ts 1100	
10	sp 350	
11	[illegible] 0	
12	0.77776	$[1-2^{-15}]$
13	0.00002	$[2^{-14}]$
14	-100	To be preset
15	-100	-(B-1) B = No. of
16	-100	√ to be calc.
17	[illegible] 0	
20	ca 1412	
21	mh 1412	
22	ts 233	
23	ca 1557	Square Re
24	mh 1557	
25	ad 233	Square Im
26	ts 233	
27	ao 520	Add
30	ao 521	
31	ao 522	
32	ao 523	
33	ao 524	
34	ao 525	
35	ao 526	
36	ao 514	
37	cp 520	
40	ca 256	
41	td 520	
42	td 525	
43	td 526	Reset
44	ca 316	
45	td 520	
46	td 521	
47	ca 317	
50	td 523	
51	td 524	
52	ca 515	
53	ts 514	
54	ca 516	
55	ts 476	
56	ca 233	
57	dv 476	
60	slr 17	
61	[illegible] 476	
62	srh 1	
63	ts 427	
64	ad 476	$\sqrt{Re^2 + Im^2}$
65	ts 476	
66	cm 427	
67	su 513	
70	cp 572	
71	sp 556	
72	ca 476	
73	ts 233	
74	ao 556	
75	ao 573	
76	ca 126	
77	ts 476	

600	ts 427	
601	ao 514	
602	cp 554	
603	ca 256	
604	td 556	
605	td 573	
606	ca 256	
07	td 610	Print control
10	ca	for magnitude
11	sp 40	
12	ao 610	
13	ao 515	
14	cp 610	
15	ca 516	sp 2130
16	ts 514	
17	ts 515	
20	ca 106	
21	rc 621	
22	rc 622	
23	sp 630	
24	0.41000	
25	0.41400	
26	[illegible] 0	+ N
27	[illegible] 0	- (N-1)
30	sp 1250	
31	[illegible] 703	(sp 670) R.I. Anm
32	ca 626	
33	ts 1100	
34	ca 625	
35	[illegible] 703	R.I. Φnm
36	ca 626	
37	ts 1245	
40	ca 624	
41	ad 626	
42	ts 624	
43	ca 625	Reset
44	ad 626	
45	ts 625	
46	ao 627	
47	cp 651	
50	[illegible] 0	Stops or [sp 2143]
51	ca 446	
52	[illegible] 707	R.O. 126→650
53	ca 447	
54	ts 125	
55	sp 1230	
56	0.21100	
57	+345	
→ 60	ca 656	1st R.S.
61	[illegible] 707	
62	ca 657	
63	ts 125	
64	[illegible] 0	
65		
66		
67		
70		
71		
72		
73		
74		
75		
76		
77		

① Phase or Phase Difference Calculation.

② Magnitude Calculation

II

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______

AUTHOR Wm. P. Wahl DATE ______

TAPE NUMBER ______

DL-481

1100	+.0000		1200	+.6249	
1101	+.0087		1201	+.6371	
1102	+.0175		1202	+.6494	
1103	+.0262		1203	+.6619	
1104	.0349		1204	.6745	
1105	.0437		1205	.6873	
1106	.0524		1206	.7002	
1107	.0612		1207	.7133	
10	.0699		10	.7265	
11	.0787		11	.7399	
12	.0875		12	.7535	
13	.0963		13	.7673	
14	.1051		14	.7813	
15	.1139		15	.7954	
16	.1228		16	.8098	
17	.1316		17	.8243	
20	.1405		20	.8391	
21	.1494		21	.8541	
22	.1584		22	.8693	
23	.1673	T	23	.8847	
24	.1763	A	24	.9004	
25	.1853	N	25	.9163	
26	.1944	G	26	.9325	
27	.2034	E	27	.9490	
30	.2126	N	30	.9657	
31	.2217	T	31	.9827	
32	.2309		32	.9999	
33	.2401	T	33	si 0	
34	.2493	A	34		
35	.2586	B	35		
36	.2679	L	36		
37	.2773	E	37		
40	.2867		40		
41	.2962		41		
42	.3058		42		
43	.3153		43		
44	.3249		44		
45	.3346		45	+0	
46	.3443		46	+.0005	
47	.3541		47	+.0010	
50	.3640		50	+.0015	
51	.3738		51	.0020	
52	.3839		52	.0025	
53	.3939		53	.0030	
54	.4040		54	.0035	
55	.4142		55	.0040	
56	.4245		56	.0045	
57	.4348		57	.0050	
60	.4452		60	.0055	
61	.4557		61	.0060	
62	.4663		62	.0065	
63	.4770		63	.0070	
64	.4877		64	.0075	
65	.4986		65	.0080	
66	.5095		66	.0085	
67	.5206		67	.0090	
70	.5317		70	.0095	
71	.5429		71	.0100	
72	.5543		72	.0105	
73	.5658		73	.0110	
74	.5773		74	.0115	
75	.5890		75	.0120	
76	.6008		76	.0125	
77	.6128		77	.0130	

NOTES

TABLE OF TANGENTS AND CORRESPONDING ANGLE TABLE USED WITH PHASE CALCULATION ROUTINE

1300	+.0135			1400	si 0	
1301	+.0140			1401	0.31100	
1302	+.0145			1402	+202	
1303	+.0150			1403	0.30346	
1304	.0155			1404	+22	
1305	.0160			1405	0.31000	
1306	.0165			1406	+51	
1307	.0170			1407	si 0	
10	.0175			10	si 0	
11	.0180			11	si 0	
12	.0185			12	ad 256	
13	.0190			13	ts 225	
14	.0195			14	ca 257	+
15	.0200			15	ts 226	
16	.0205			16	sp 143	SINE
17	.0210			17	mr 260	SIGN
20	.0215			20	sl 17	
21	.0220	A		21	ts 225	ROUTINE
22	.0225	N		22	ca 257	+
23	.0230	G		23	ts 226	
24	.0235	L		24	sp 143	
25	.0240	E		25	ad 256	
26	.0245			26	ts 225	
27	.0250	T		27	ca 261	−
30	.0255	A		30	ts 226	
31	.0260	B		31	sp 143	
32	.0265	L		32	mr 260	
33	.0270	E		33	sl 17	
34	.0275			34	ts 225	−
35	.0280			35	ca 261	
36	.0285			36	ts 226	
37	.0290			1437	sp 143	
40	.0295			1440	si 0	
41	.0300		1441			
42	.0305					
43	.0310					SINE TABLE
44	.0315					STORAGE
45	.0320					
46	.0325		1523			
47	.0330		1524		si 0	
50	.0335		1525		ca 1401	
51	.0340				sp 207	Ro. TANGENTS AND
52	.0345				ca 1402	ANGLES
53	.0350				ta 1400	
54	.0355				ca 1403	
55	.0360				sp 207	Ro. SINE SIGN
56	.0365				ca 1404	
57	.0370				ta 1402	
60	.0375				ca 1405	
61	.0380				sp 207	Ro. SINE VALUES
62	.0385				ca 1406	
63	.0390				ta 1404	
64	.0395		1544		si 0	
65	.0400			65		
66	.0405			66		
67	.0410			67		
70	.0415			70		
71	.0420			71		
72	.0425			72		
73	.0430			73		
74	.0435			74		
75	.0440			75		
76	.0445			76		
77	.0450			77		

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ________ INDEX ________
AUTHOR Ma C Welsh DATE ________
TAPE NUMBER 3436 M3

DL-481

Fig. (1)

Address	Instruction	Remarks
00		
01		
02		
03		
04		
05		
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56		
57		
60		
61		
62		(CONTINUED)
63		
64		
65		
2166	ao 550	
2167	qp 2177	
2170	ca 176	
2171	ca 106	
2172	ac 2172	
73	ac 2173	
74	ac 2174	
75	ca 124	
76	qp 2111	→ out
77	ca 176	

Address	Instruction	Remarks
2200	ca 244	
2201	ac 2201	
2202	ca 246	
2203	ac 2203	
2204	ac 2204	
2205	ca 245	
2206	ac 2206	
07	ca 24	
10	ac 2210	
11	qp 2121	
12	ao 2223	
13	qp 2227	
14	ca 2225	CHANGE SPECTRUM
15	td 2124	GROUPS
16	ca 2226	
17	td 2125	
20	ca 2224	
21	td 2136	
22	qp 2141	
23	-23	PRESET!
24	ca 2124	
25	0.05100	NEW AMPLITUDE GROUP
26	0.11100	" PHASE "
27	ca 2134	
30	qp 2137	
31		
32		
33	+0	
34	+0	
35	+.0450	
36	+0	
37	-40	-30
40	-40	-30
41	0.02500	
42	-7	
43	-7	
44	+.0036	+.0360
45	+.0100	+1000
46	-1	
47	+100	+10
50		
51	ao 2246	
52	qp 2254	
53	qp 2257	
54	ca 2235	
55	mh 2245	
56	td 2235	
57	ca 2233	
60	mh 2235	
61	sl 17	
62	mh 2234	
63	sl 17	
64	td 2500	
65	ao 2234	
66	ao 2236	
67	ao 2237	
70	qp 2257	
71	ca 2236	
72	td 2234	
73	ca 2241	
74	td 2264	
75	ca 2240	
76	td 2237	
77	ao 2233	

NOTES

Address	Instruction	Remarks
2300	ao 2242	
2301	qp 2306	
2302	td 2236	
2303	td 2233	
2304	ca 2243	
2305	td 2242	
2306	ca 2500	
07	qp 2313	
10	mh 2244	
11	td 2500	PUT
12	qp 2306	δ_K
13	ad 2244	INTO
14	td 2500	FORM
15	ao 2306	0 ··· .0036
16	ao 2311	
17	ao 2314	
20	ao 2237	
21	qp 2306	
22	ca 2240	
23	td 2237	
24	ca 2241	
25	td 2306	
26	td 2311	
27	td 2314	
30	ca 2500	
31	mh 2247	
32	sl 17	
33	td 2500	
34	ao 2330	FACTOR
35	ao 2333	BY
36	ao 2237	100
37	qp 2330	
40	ca 2241	
41	td 2330	
42	td 2333	
43	ca 2240	
44	td 2237	
45	ca 233	
46	qp 2355	
47	ao 2345	
50	ao 2355	PUT PHASE
51	ao 2357	IN +0→+.3600
52	ao 514	RANGE
53	qp 2345	
54	qp 2361	
55	ca 233	
56	ad 2126	
57	td 233	
60	qp 2347	
61	ca 256	
62	td 2345	
63	td 2355	RESET
64	td 2357	
65	ca 515	
66	td 514	
67	qp 2430	
70	mh 2500	
71	td 233	ADD
72	ao 2450	PHASE
73	ao 2371	SHIFT
74	ao 2453	δ_K
75	qp 2416	
76	ca 256	
77	td 2450	

CALCULATION OF PHASE SHIFT
(A FUNCTION OF POSITION AT TABLE
AND FREQUENCY)

	Address	Instruction	Remarks
	2400	td 2371	
	2401	ca 2454	
	2402	td 2453	
	2403	ca 251	
	2404	qp 2461	qp 2420 -
	2405	ao 2414	
	2406	qp 2411	
	07	ca 2065	
	10	qp 2146	→ RESET 627
	11	ca 2415	FOR ANOTHER
	12	td 627	AY SPECTRUM
	13	qp 651	
	14	-1	
	15	-23	PRESET!
	16	ao 2370	
	17	qp 2452	
	20	ca 2241	
	21	td 2370	
	22	ca 251	
	23	qp 2411	→ PRINT PHASE
	24	-7	
	25	-7	
	26	-9	
	27	-9	
	30	ca 2500	
	31	td 2510	
	32	ao 2430	
	33	ao 2431	
	34	ao 2424	
	35	qp 2430	
	36	ca 2241	
	37	td 2430	
	40	ca 2425	
	41	td 2424	
	42	ao 2427	
	43	qp 2430	
	44	ca 2426	
	45	td 2427	
	46	ca 2452	
	47	td 2431	
	50	ca 233	
	51	qp 2370	or qp 2376 TO AVOID
	52	0.04510	ADDING δ_K
	53	-40	TAKE PRESET
	54	-40	
	55		
	56		
	57		
402		ca 176	
111		-19	
112		-19	
222		qp 232	CHANGES IN AMPLITUDE
400		qp 411	PROGRAM
1067		mh 757	
2074		qp 2103	
2117		qp 500	
2136		qp 2112	
2143		qp 2445	
	72		
	73		
	74		
	75		
	76		
	77		

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm P Wolf DATE ______
TAPE NUMBER ______

DL-481

	Address	Content	Remarks
		0.31000	
	1400	0.04000	— Group 6 — Random
	1401	+ 40	+ N' (PRESET)
	1402	+ 10	+ K (PRESET) SKIP
	1403	+ 0	
	1404	+ 1	(−0)
	1405	+ 0	
	1406	0.02000	
	07	0.02000	
	10	0.02400	
	11	−1	
	12	−40	Multiplying / PRESET
	13	−40	counters / TO
	14	−40	2nd counter / N'
	15	−40	
	16	−40	Factor counter
	17	0.02000	
	20	+ 0	
	21	0.02401	
	22	−200	
	23	−200	
	24	−48	−M
	25	−48	−M
	26	−24	
	27	−24	
	30	0.24100	Spectrum
	31	0.05100	Groups
	32	+ 41	
	33	ar 1424	Print M
	34	sp 1440	spectra
	35	ca 1425	
	36	ts 1424	
	37	ck 0	[Stops]
START	40	ca 1400	
	41	ar 703	R.I. Data
	42	ca 1401	
	43	ts 2000	
	44	ca 1400	
	45	ad 1402	
	46	ts 1400	Set Skip
	47	ca 2000	
	50	mh 2000	
	51	ad 2400	
	52	ca 1403	
	53	ar 1404	(sp 3300)
	54	sp 1456	
	55	sp 1520	— Factoring Routine
	56	ca 2000	
	57	mh 2000	
	60	ad 2400	
	61	ts 2400	
	62	ar 1447	
	63	ar 1450	
	64	ar 1456	
	65	ar 1457	
	66	ar 1413	
	67	sp 1447	
	70	ar 1412	Set 4mb
	71	ts 1413	counter
	72	ar 1451	Shift
	73	ar 1460	Down
	74	ar 1461	
	75	ar 1407	set up
	76	td 1450	storage
	77	td 1457	for R.R. R. Ra

	Address	Content	Remarks
	1500	ca 1406	
	1501	td 1447	
	1502	td 1456	
	1503	ar 1414	
	1504	sp 1447	→ Calculate R. R. Ra
	1505	ca 1406	Reset
	1506	ts 1407	Shift
	07	ca 1410	
	10	td 1451	Reset
	11	td 1460	for
	12	td 1461	R. Storage
	13	ca 1415	
	14	ts 1412	Reset
	15	ts 1413	counters
	16	ts 1414	(sp 3343)
	17	sp 1550	→ Spectrum Factoring
	20	ca 1405	
	21	ts 1403	
	22	ca 2000	
	23	ad 2401	
	24	ts 2000	
	25	ar 1522	
	26	ar 1524	Data
	27	ar 1416	
	30	sp 1522	Factoring
	31	ca 1415	Reset
	32	ts 1416	counter / Routine
	33	ca 1406	
	34	td 1522	
	35	td 1524	
	36	td 1447	
	37	td 1450	
	40	td 1456	
	41	td 1457	
	42	ca 1415	Reset
	43	ts 1413	counter
	44	sp 1447	→ Try once again (sp 3300)
	45	+.5000	
	46	−1	
	47	0.00401	
	50	ca 2400	
	51	sp 1561	
	52	ts 2000	Spectrum
	53	ar 1550	
	54	ar 1552	Factoring
	55	ar 1561	
	56	ar 1416	Routine
	57	sp 1550	
	60	sp 1565	
	61	ca 2400	
	62	mh 1411	Change
	63	sl 17	⊖ value to ⊕
	64	sp 1552	
	65	ca 1415	Reset
	66	ts 1416	counter
	67	ca 1410	
	70	td 1550	0.02400
	71	td 1561	
	72	ca 1417	
	73	td 1552	0.02600
	74	ca 1400	
	75	ad 2400	
	76	ca 1403	
	77	su 1404	

NOTES

	Address	Content	Remarks
	1600	sp 1602	
	1601	sp 1621	→ Factor a_o
	1602	ca 1420	
	1603	ad 2000	
	1604	ts 1420	
	1605	ar 1603	
	1606	ar 1575	
	1607	ar 1416	
	10	sp 1574	→
	11	ca 1415	Reset
	12	ts 1416	counter
	13	ca 1417	
	14	td 1575	0.02400
	15	td 1603	
	16	ca 1405	+ 0
	17	ts 1420	
	20	sp 1655	→ Restore R's
	21	ca 1405	
	22	ts 1403	
	23	ca 2400	
	24	ad 2401	
	25	ts 2400	Factor
	26	ar 1623	Spectrum
	27	ar 1625	Data
	30	ar 1414	(R. R. Ra)
	31	sp 1623	
	32	ca 1410	
	33	td 1623	
	34	td 1625	
	35	ca 1415	
	36	ts 1414	
	37	ts 1416	
	40	ar 1576	
	41	sp 1646	→
	42	ca 1411	Reset to +1
	43	ts 1546	
	44	sp 1574	
	45	ca 1405	Reset to
	46	ts 1420	+ 0
	47	ca 1417	
	50	td 1623	
	51	td 1625	
	52	td 1575	
	53	td 1603	
	54	sp 1623	→ Symmetry
	55	ca 2400	ca 1545
	56	mh 1545	Factor a_o / mh 1737
	57	ts 400	6Y + 5 / ts 400
	60	ca 2401	Transfer
	61	ts 401	to new storage
	62	ar 1660	
	63	ar 1661	
	64	ar 1416	
	65	sp 1660	
	66	ca 1411	
	67	td 1660	
	70	ca 1547	
	71	td 1661	
	72	ca 1415	Reset
	73	ts 1416	counter
	74	ca 0	
	75	ts 2400	Print
	76	ar 1675	in R storage
	77	ar 1422	

	Address	Content	Remarks
	1700	sp 1674	
	1701	ca 1410	0.02400
	1702	td 1675	
	1703	ca 1423	Reset
	1704	ts 1422	counter
	05	sp 143	→ Calculate Power Spectrum
	06	ar 1444	
	07	sp 1711	
	10	sp 1721	→
	11	ts 1430	
	12	ar 707	R.O. Spectrum
	13	ca 1730	
	14	ts 1440	
	15	ca 1430	Set for
	16	ad 1432	next
	17	ts 1430	spectrum
	20	sp 1740	→ Begin again →
	21	ca 1427	Reset
	22	ts 1426	counter
	23	ca 1736	
	24	td 1707	(Element)
	25	ca 1431	
	26	ar 707	R.O. Spectrum
	27	ca 1432	
	30	ts 1440	
	31	ca 1431	31
	32	ad 1432	
	33	ts 1431	
	34	sp 1071	→ Begin again →
	35	0.05100	
	36	0.01725	
	37	+.1000	
	40	sp 1071	
	41		
	42		
	43		
	44		
	1745		
	1746		
	1747		
	50		
	51		
	52		
	53		
	54		
	55		
	56		
	57		
125	60	sp 126	Changes in
130	61	sp 1433	Power Spectrum
224	62	sp 274	Program
1062	63	sp 1706	
	64		
	65		
	66		
	67		
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	74		
	75		
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	77		

AUTOCORRELATION — ADAPTED
FOR POWER SPECTRUM
AND DENSITY PLOT ROUTINE

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm. P. Walsh DATE ______
TAPE NUMBER 106-43-05021

M18. (J)

DL-481

52

Fig. (K)

	Address	Instruction	Remarks
	3000	ao 3133	
	3001	sp 3003	
*	3002	ck 0	Stop Here!
	3003	ca 1427	
	3004	ta 1426	
	3005	ca 3110	
	3006	td 1207	
	3007	ca 3127	
	10	ta 1430	Set up
	11	ca 3130	Down Data
	12	ta 1431	Storage
	13	ca 1033	
	14	ta 3113	
	15	ca 3114	
	16	ta 1400	
	17	ca 3115	
	20	ta 1032	
	21	ca 3116	
	22	sr 707	R.C. Program
	23	ca 3117	
	24	sr 1050	
	25	sp 31	→ R3 Tape
	26	ca 3113	
	27	ta 1033	
	30	ca 1055	
	31	sr 1057	
	32	mh 3102	Subtract
	33	slh 17	Mean
	34	ta 1055	Factor
	35	ao 3030	
	36	ao 3034	
	37	ao 3121	
	40	cp 3030	
	41	ca 3122	
	42	ts 3121	
	43	ca 3123	
	44	td 3034	
	45	td 3030	
	46	ca 3114	
	47	sr 707	R.O. Data
	50	ca 3124	
	51	sr 1055	
	52	ca 3116	
	53	sr 703	R.I. Program
	54	ca 3117	
	55	sr 1050	
	56	ca 0	
	57	ta 2000	
	60	ao 3057	Put 0 in
	61	ao 3126	Data Storage
	62	cp 3056	
	63	ao 3125	Reset
	64	ts 3126	Counter
	65	ca 1406	
	66	td 3057	
	67	ca 3111	(or ca 3130)
	70	ta 3127	Set up new
	71	ca 3112	Down Storage
	72	ta 3130	
	73	sp 1433	→ Calc. - Autocor.
* S.A.	74	ca 3131	
	75	ta 42	
	76	ta 125	
	77	ca 3132	

	Address	Instruction	Remarks
	3100	ta 1031	
	3101	sp 3007	→
	3102	ca 3105	
	3103	ta 3142	
*	3104	sp 3024	→
	3105	ca 0	
	3106		
	3107		(-M) Tape Preset
	10	+01711	
	11	0.14100	②
	12	0.11100	①
	13	ck 0	
	14	0.07000	0.31000
	15	sp 3006	
	16	0.20000	
	17	+466	
	20	+0010	
	21	-540	
	22	-540	
	23	0.01055	
	24	+541	
	25	-100	
	26	-100	
	27	0.24100	①
	30	0.05100	②
	31	sr 405	
	32	sp 1014	
	33	-1	
	34	sp 3142	
	35	-984	
	36	+984	
	37	0.05100	
S.A.	40	ca 3134	2 Pictures / 80 L 10 PT skip / 2 Tapes
	41	ta 3000	
	42	sp 3102	
	43	ta 40	
	44	ao 3143	
	45	ao 3135	
	46	sp 3142	S ca 3112
	47	ca 3112	
	50	sr 707	1st Picture
	51	ca 3136	Blank
	52	ta 40	
	53	ao 3137	(or ca 3111 3rd = 0)
	54	sr 707	2nd Picture
	55	ca 3136	Blank
	56	ta 40	
*	57	ck 0	Stop Here
	60	ca 344	
	61	sp 40	
263	62	ck 134	Changes in
264	63	sp 374	Autocorrelation
1437	64	sp 3000	Program
	65		
S.A.	3166	ca 3160	Direct
	3167	ta 224	Printer
	70	ca 3161	
	71	ta 1062	
	72	sp 3024	→
S.A.	73	ca 3160	Delayed
	74	ta 224	Printer
	75	ca 3161	
	76	ta 1062	
	77	sp 3007	→

NOTES Data Factoring ~ Read In ~ Output Control

I 4 Pictures [2 40 L Tapes ~ 2 80 L Tapes (10 PT. skip)] S.A. 3074
II 2 " [1 80 L " ~ 10 PT skip] S.A. 3267
III 2 " [2 80 L " ~ 20 PT skip] S.A. 3261

	Address	Instruction	Remarks
	3200	ca 3210	Change Counter
	3201	ta 733	in 80 Loc
	3202	ta 734	
	3203	ca 0	
	3204	ta 130	
	3205	ca 3211	* Start at 1400
	3206	ta 1402	after each calculation
	3207	sp 3166	→
	10	-79	
	11	+80	
	12	ca 0	
	13	ta 1402	
	14	ta 130	
	15	sp 3217	* Start at 3007
	16	td 1	after each calc.
	17	ca 3210	
	20	ta 733	
	21	ta 734	
	22	sp 3166	→
	23	ca 3231	
	24	ta 3207	
	25	sp 3200	→ Same as above
	26	ta 3231	save Delayed
	27	ta 3232	Counter is used
	30	sp 3204	→
	31	sp 3173	
S.A.	32	ca 0	
	33	ta 1435	
	34	ca 3210	
	35	ta 733	80 L
	36	ta 734	
	37	ca 3107	Set to
	40	ta 1424	of Spectra
	41	sp 3166	→ Direct Print
S.A.	42	ta 3231	
	43	ta 3241	
	44	sp 3232	→ Delayed Print
	45	+20	
S.A.	46	ca 3256	
	47	ta 3210	
	50	ca 3257	
	51	ta 3211	40 L
	52	sp 3222	→ Direct Print
S.A.	53	ca 3260	
	54	ta 3252	
	55	sp 3246	→ Delayed Print
	56	-39	
	57	+40	
	60	sp 3272	
S.A.	61	ta 3245	
	62	ta 1400	
	63	ao 1426	1 Picture
	64	ta 1424	Out Put
	65	ta 1425	Control 80 L
	66	sp 3140	
S.A.	67	ca 0	
	70	ta 3133	
	71	ca 3111	
	72	ta 3137	
	73	sp 3140	
	74		
	75		
	76		
	77		

Print Out Control (40 & 80 L)

Direct ~ {40 L — S.A. - 3276; 80 L — S.A. - 3200} Set
Delayed ~ {40 L — S.A. - 3253; 80 L — S.A. - 3242} 3107

	Address	Instruction	Remarks
	3300	sp 1520	→ -1 overflow
	3301	ad 1404	
	3302	sp 1556	→ no overflow
	3303	sp 1520	→ +1 overflow
	3304	ta 1414	
	3305	ta 1412	
	3306	ca 1410	
	3307	td 1451	
	10	td 1460	
	11	td 1461	
	12	ca 0	
	13	ta 2400	
	14	ao 3313	
	15	ao 1416	
	16	sp 3312	
	17	ca 1415	Normalizing
	20	ta 1416	and
	21	ca 1410	Factoring
	22	td 3313	Routine
	23	sp 1447	→
	24	ca 2401	
	25	dv 2400	
	26	slh 17	
	27	mh 1237	
	30	ta 2401	
	31	ao 3324	
	32	ao 3330	
	33	ao 1416	
	34	sp 3324	
	35	ao 1415	
	36	ta 1416	
	37	ca 1421	
	40	td 3324	
	41	td 3330	
	42	sp 1455	→
	43	ca 2400	
	44	ad 1404	
	45	sp 3347	sp(a) Zero
	46	sp 3304	check
	47	ca 2400	
	50	ta 400	
	51	sp 1660	
	52		
	53		
	54		
	55		
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MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm. P. Walsh DATE ______
TAPE NUMBER 10643-05023

DL-481

Fig. (1)

2100	ca 1055	
2101	su 1054	Subtract
2102	mh 2200	Mean
2103	sh 17	
2104	ts 40	Scale
2105	ao 2100	Factor
2106	ao 2104	
07	ao 2211	
10	cp 2100	
11	ca 2201	
12	td 2114	
13	td 2116	
14	ca	Factor
15	mh 2202	R_0 & R_{40}
16	ts	
17	ca 2204	
20	ad 2215	
21	ts 2201	
22	ao 2212	
23	cp 2111	
24	ca 2213	
25	ts 2212	
26	ca 2203	Ro. factored data
27	su 207	
30	ca 2210	
31	ts 40	
32	ca 2217	Shift 10, -20, -30, -40
33	ad 2216	
34	ts 2201	
35	ca 2220	
36	td 2100	Reset
37	ca 2217	
40	td 2104	
41	ca 2214	
42	ts 2211	
43	ao 2126	
44	ca 2216	Set for another ten
45	ad 2221	
46	ts 2216	
47	ao 2207	
50	cp 2100	
51	sp 0	Stop
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2200	+.0010	
2201	0.00040	
2202	+.5000	
2203	0.34100	
2204	0.41100	
2205	0.04100	
2206	0.10100	
07	-3	
10	+511	
11	-510	
12	-12	
13	-12	
14	-510	
15	+40	
16	+10	→ 10 pt. skip
17	0.00040	
20	0.01055	
21	+10	
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NOTES

40 Lag Trace (Data)
Preparation
Routine

2200	+.0010	
2201	0.02300	
2202	+.5000	~~ampl.~~ ~~[illegible]~~
2203	0.34100	7 - 100
2204	0.41100	1 - 400
2205	0.04100	1 - 100
2206	0.10100	1 - 100
07	-3	
10	+541	
11	-540	
12	-6	
13	-6	
14	-540	
15	+80	
16	+20	→ 20 pt. skip
17	0.02300	
20	0.01055	
21	+20	
22	ca 1055	Subtract
23	su 1054	Mean
24	mh 2200	
25	sh 17	Scale
26	ts 4300	Factor
27	ao 2222	
30	ao 2226	
31	ao 2211	
32	cp 2222	
33	ca 2201	
34	td 2236	
35	td 2240	Factor
36	ca	R_0 & R_{80}
37	mh 2202	
40	ts	
41	ca 2201	
42	ad 2215	
43	ts 2201	
44	ao 2212	
45	cp 2233	
46	ca 2213	
47	ts 2212	
50	ca 2203	Ro. data
51	su 207	
52	ca 2210	
53	ts 4300	
54	ca 2217	Shift 20, 40, 60, 80
55	ad 2216	
56	ts 2201	
57	ca 2220	
60	td 2222	
61	ca 2217	
62	td 2226	
63	ca 2214	
64	ts 2211	
65	ao 2250	
66	ca 2216	
67	ad 2221	
70	ts 2216	
71	ao 2207	
72	cp 2222	
73	sp 0	
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80 Lag Trace (Data)
Preparation
Routine

106-93-06001

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MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______
AUTHOR Wm P Wahl DATE ______
TAPE NUMBER ______

DL-481

Fig. (m)

Address	Instruction	Remarks
460	ca 560	RI ~ Re ~ or ϕ_{mn}
461	ex 203	
462	ca 552	
463	sf 1412	
464	ca 560	=> [ca 561 => (for cross-spectrum)]
65	ex 703	RI ~ Im ~ or ϕ_{mn}
66	ca 552	
67	sf 1557	
70	ao 544	
71	cp 473	
72	sp 502	
73	ca 561	setup second overlap
74	ad 553	
75	ts 561	
76	ca 555	
77	td 460	

Address	Instruction	Remarks
500	td 464	
501	sp 564	
502	ao 545	
503	cp 505	
504	sp 514	
505	ca 562	set up third overlap
506	ad 553	
07	ts 562	
10	ca 556	
11	td 460	
12	td 464	
13	sp 564	
14	ao 546	
15	cp 517	
16	sp 526	
17	ca 563	setup fourth overlap
20	ad 553	
21	ts 563	
22	ca 557	
23	td 460	
24	td 464	
25	sp 564	
26	sp 527	
27	ca 551	shift 40 down
30	ts 553	
31	ca 560	
32	ad 553	
33	ts 560	
34	ca 554	set up fifth overlap
35	td 460	
36	td 464	
37	ca 547	
40	ts 544	
41	ts 545	
42	ts 546	
43	sp 564	
44	−1	
45	−1	
46	−1	
47	−1	
50	−1	
51	+40	To be preset [+(N−1)]
52	+41	[+N]
53	+0	
54	0 00560	
55	0 00561	
56	0 00562	value
57	0 00563	80 lag
60	0.34100	To be preset 0 34100
61	0.41112	0 41424
62	0.04124	0 04150
63	0 10136	0 10174
64	sp 2166	→ out

NOTES

① Read in routine of prepared data (trace or cross-correlation)

② Above used with 40 lag data

Address	Contents
40	+ 0010
41	0 02400
42	+ 5000
43	0 34100
44	0 41100
45	0 26100
46	0.16100
47	−3
50	+676
51	−675
52	−6
53	−6
54	−675
55	+100
56	+25
57	0 02400
60	0 01055
61	+25

Address	Instruction	Remarks
100	ca 1055	
101	su 1054	
102	mh 40	Subtract Mean
103	sl 17	
104	ts 2400	
105	ao 100	Scale Factor
106	ao 104	
107	ao 51	
10	cp 100	
11	ca 41	
12	td 114	
13	td 116	Factor $R_0 \sim R_{100}$
14	ca	
15	mh 42	
16	ts	
17	ca 41	
20	ad 55	
21	ts 41	
22	ao 52	
23	cp 111	
24	ca 53	
25	ts 52	
26	ca 43	
27	sr 707	R.O. Data
30	ca 50	
31	bo 2400	
32	ca 57	
33	ad 56	Shift 25
34	ts 41	
35	ca 60	
36	td 100	
37	ca 57	
40	td 104	
41	ca 54	
42	ts 51	
43	ao 126	
44	ca 56	
45	ad 61	
46	ts 56	
47	ao 47	
50	cp 100	
51	sr 0	

Data Factoring Routine

100 Lags

106-43-06021

MIT DIGITAL COMPUTER LABORATORY
OCTAL PROGRAM FORM

TITLE ______ INDEX ______

AUTHOR Wm. C. Walsh DATE ______

TAPE NUMBER ______

DL-481

Appendix C

APPENDIX C

Weston Observatory -- Pertinent Data

Geodetic coordinates:	42° 23' 04.9" N
	71° 19' 19.5" W
Elevation:	60 meters
Lithologic foundation:	Metavolcanic
Pendulum mass:	100 kg.
Instruments:	Vertical, N-S, E-W, Benioff long and short period seismographs.

Normal Operating Constants:

Instrument	T_o sec.	T_g sec.	Drum Speed
ZSP	1.0	0.5	60 mm/min
NSP	1.0	0.25	60
ESP	1.0	0.25	60
ZLP	1.0	30.0	30
NLP	1.0	60.0	30
ELP	1.0	60.0	30

T_o -- Period of pendulum.

T_g -- Period of galvanometer.